

# A Model to Analyse Financial Fragility: Applications<sup>\*†</sup>

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## Abstract

The purpose of our work is to explore contagious financial crises. To this end, we use simplified, thus numerically solvable, versions of our general model [Goodhart, Sunirand and Tsomocos (2003)]. The model incorporates heterogeneous agents, banks and endogenous default, thus allowing various feedback and contagion channels to operate in equilibrium.

Such a model leads to different results from those obtained when using a standard representative agent model. For example, there may be a trade-off between efficiency and financial stability, not only for regulatory policies, but also for monetary policy. Moreover, agents which have more investment opportunities can deal with negative shocks more effectively by transferring ‘negative externalities’ onto others.

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# 1 Introduction

The purpose of our work is to explore contagious financial crises. To this end we need a model of heterogeneous banks with differing portfolio; if all banks were identical, or there was only one bank, there would be no interbank market and no contagion by definition. We also require a set-up in which default exists, and can be modelled. Otherwise there would be no crises. Similarly financial markets cannot be complete, since, if there were, all eventualities could be hedged. Finally, since we are concerned with financial crises, there must be an inherent role for money, banks and interest rates. We have constructed a rational expectations, forward-looking dynamic general equilibrium model along these lines in Goodhart et al. (2003).

Default is modelled as in Shubik and Wilson (1977). By varying the penalties imposed on default from 0 to infinity, we can model 100% default (0 penalty), no default (infinite penalty) or an equilibrium default level between 0 and 100%. The main financial imperfection is that we assume that individual bank borrowers are assigned during the two periods of our model, by history or by informational constraints, to borrow from a single bank.<sup>1</sup> Money is introduced by a cash in advance constraint, whereby a private agent needs money to buy commodities from other agents; commodities cannot be used to buy commodities. Similarly we assume that agents needing money can always borrow cheaper from their (assigned) bank than from other agents; banks have an informational (and perhaps scale) advantage that gives them a role as an intermediary.

In our general model (Goodhart, Sunirand and Tsomocos (2003)) there are a set of heterogeneous private sector agents with initial endowments of both money and commodities; it is an endowment model without production. There is also a set of heterogeneous banks, who similarly have differing initial allocations of capital (in the form of government bonds)<sup>2</sup>. There are two other players, a Central Bank which can inject extra money into the system, e.g. by buying an asset or lending, and a Financial Supervisory Agency, which can set both liquidity and capital minimum requirements and imposes penalties on failures to meet such requirements and on defaults. We do not seek to model the actions of these official players. They are strategic dummies.

The game lasts two periods. Period one involves trading in bank loans, bank deposits (including interbank deposits), a potential variety of other financial assets, e.g. an Arrow-type security or bank equity, and commodities. Such trading is done in anticipation that nature will randomly select a particular state,  $s \in S = \{i, ii, \dots, S\}$ . In period 2, dependent on the state actually selected, there is further trading in commodities; all loans, including interbank loans, are repaid, subject to any defaults, which are then penalised, and the banks are in effect wound-up. The timeline of this model is shown in Figure 1.

In Goodhart et al. (2003) we demonstrate that such a model has an equilibrium and can be solved. We show that financial fragility emerges naturally as an equilibrium phenomenon. In our model financial fragility is characterised by reduced aggregate bank profitability and increased aggregate default as in Tsomocos (2003a and b). Whenever such financial fragility is present in the economy, the role of economy policy is justified. Regulatory and monetary policies are shown to be non-neutral due to the lack of the classical dichotomy between the real

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<sup>1</sup>Restricted participation can also arise as an outcome of banks aiming to outperform each other by introducing a relative performance criterion into their objective functions. For more on this see Bhattacharya, Goodhart, Sunirand and Tsomocos (2003).

<sup>2</sup>Commercial banks are modelled as in Shubik and Tsomocos (1992). The modelling of banks is akin to Tobin (1962 and 1982).

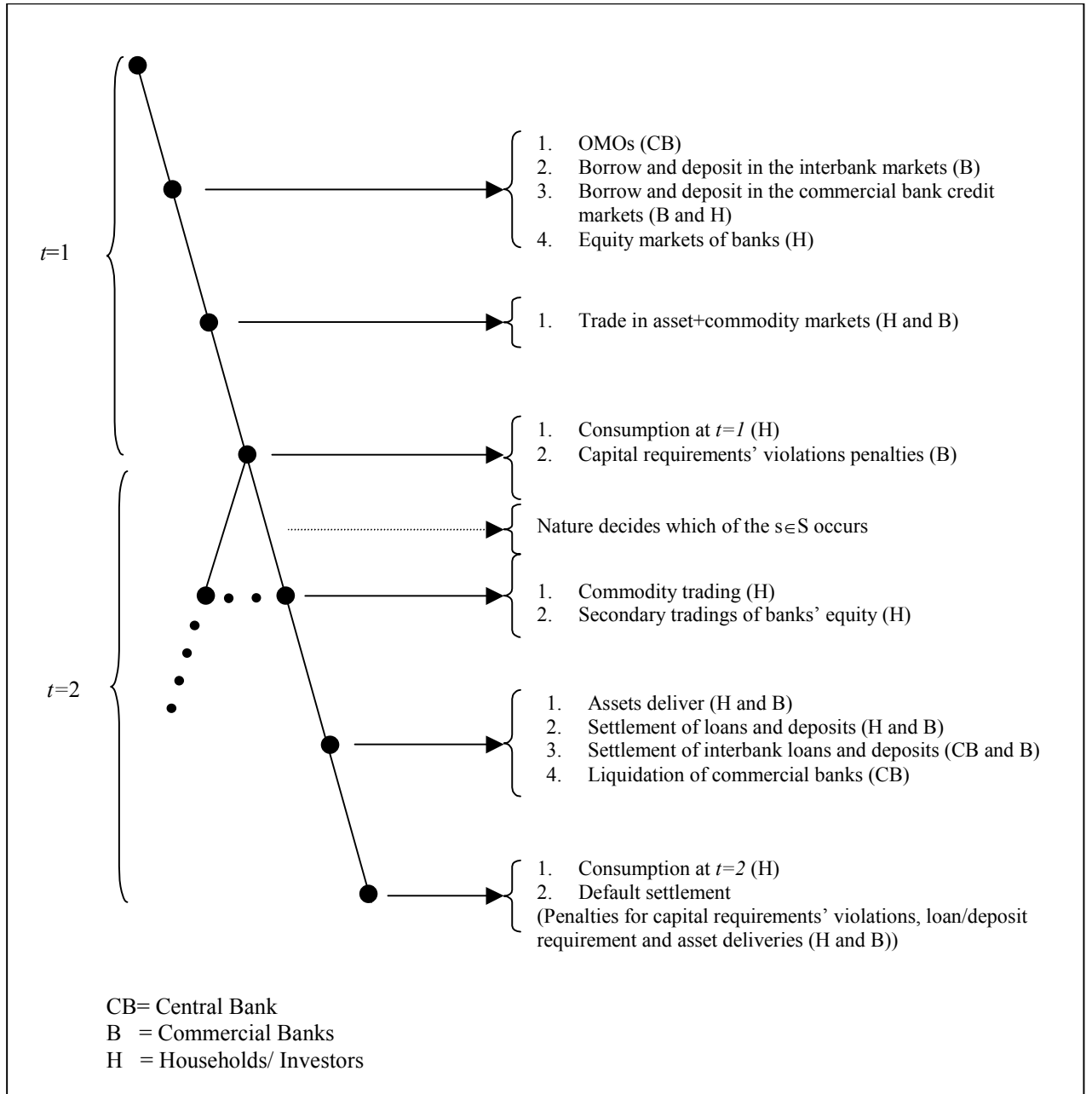


Figure 1: The The Structure of the Model

and nominal sectors of the economy. We also show that a non-trivial quantity theory of money holds, and the liquidity structure of interest rates depends both on aggregate liquidity and the risk of default in the economy. Finally, we address formally the Modigliani-Miller proposition, and establish the conditions that cause its failure. In particular, it fails *either* due to limited participation *or* incomplete (financial) markets *or* different risk preferences among banks. Given the scale of the model with  $b$  heterogeneous banks,  $n$  private sector agents,  $S$  states, a variety of financial assets, default and default penalties, and a variety of non-linearities, it is impossible to find either a closed-form or a numerical solution to the general model. The purpose of this paper, therefore, is to present a smaller, specific version of this model which can be numerically solved.

## 2 The Base-line Model

We first simplify the general model fully developed in Goodhart et al. (2003) to the case of three households,  $h \in H = \{\alpha, \beta, \phi\}$ , and two banks,  $b \in B = \{\gamma, \delta\}$ , with a Central Bank which conducts monetary policy through open market operations (OMOs) and a regulator, which fixes the bankruptcy code for households and commercial banks as well as sets the capital-adequacy requirements for banks. The decisions of households and banks are endogenous in the model, whereas the Central Bank and the regulator are treated as strategic dummies with pre-specified strategies. The time horizon extends over two periods ( $t \in T = \{1, 2\}$ ) and three possible states ( $s \in S = \{i, ii, iii\}$ ) in the second period.

Given the cash-in-advance constraint, money is essential in the model. There are 4 active markets in this economy: commodity, consumer credit, interbank, and financial asset markets. In period 1, the commodity, asset, credit and interbank markets meet. At the end of this period consumption and settlement, including any bankruptcy and capital requirements' violation penalties, take place. In period 2, the commodity market opens again, loans are settled and assets are delivered. At the end of this period consumption and settlement for default and capital requirements' violations take place. Also, commercial banks are liquidated.

In order to show inter-connections between banks, we need at a minimum two banks. One bank, bank  $\gamma$ , is relatively poor at  $t = 1$  and therefore has to seek external funds to finance its loans. As in the general model, we assume a limited participation assumption in the consumer loan market. Thus, bank  $\gamma$  lends to its *nature-selected* borrower, Mr.  $\alpha$ . Bank  $\gamma$  can raise its funds either by borrowing from the *default free* interbank market<sup>3</sup> or selling its securities. In general, there are a variety of financial assets, besides deposits and bank loans, that we can introduce into the model, but owing to the size of the system, amounting to over 60 equations, we can only do so one at a time. In our first base-line model, we introduce an Arrow-type security, which the weaker bank (bank  $\gamma$ ) can issue. This pays out 1 in state  $i$  in period 2, and nothing in any other states. In this way, state  $i$  is regarded as the 'good' state whereas the other two states, states  $ii$  and  $iii$ , are treated as 'bad' states.<sup>4</sup> Bank  $\gamma$  can be thought of as a typical straightforward small commercial bank. Its assets comprise only its credit extension to the consumer loan market. This way we can focus on the effects of policies on banks that cannot quickly restructure their portfolios by diversifying their asset investments, perhaps due to inaccessibility of capital and asset markets, during periods of financial adversity. The other

<sup>3</sup>In section 3, we relax this assumption, allowing default both in the interbank and deposit markets.

<sup>4</sup>Note that since there are three states and two assets (loans and the Arrow security), markets are incomplete and therefore equilibria are constrained inefficient. Thus, there is scope for welfare improving economic policy (both regulatory and monetary).

bank, bank  $\delta$ , is a large and relatively rich investment bank which, in addition to its lending activities to its *nature-selected* borrower, Mr.  $\beta$ , has a portfolio consisting of deposits in the interbank market and investment in the asset market (i.e. purchasing bank  $\gamma$ 's Arrow security). Its richer portfolio allows it to diversify quickly and more efficiently than bank  $\gamma$ . As we shall see later, this extended opportunity set enables bank  $\delta$  to transfer the negative impact of adverse shocks to the rest of the economy without necessarily reducing its profitability.

Given our restriction that agents can borrow only from a single bank, we need three agents, two who want to borrow in period 1 (Mr.  $\alpha$  and  $\beta$ ), because they are comparatively more constrained in money and goods in period 1 relative to period 2, and want to smooth consumption over time. The third agent, Mr.  $\phi$ , is richer in both goods and money in period 1, relative to period 2, and hence deposits money with the banks in period 1 and sells goods to the borrowers. He deposits money with bank  $\gamma$ , which in equilibrium offers the highest *default free* deposit rate, and buys Arrow securities to transfer wealth from  $t = 1$  to  $t = 2$ , and thus smooth his consumption. In a sense, Mr.  $\alpha$  and  $\beta$  represent the household sector of the economy in which their main activity is borrowing for present consumption in view of future expected income. On the other hand, Mr.  $\phi$  represents the investors' sector, with a more diversified portfolio consisting of deposits and investments in the asset market, in order to smooth his intertemporal consumption. At this stage we assume that the deposit rate is always equal to the lending rate offered by bank  $\gamma$  i.e. perfect financial intermediation.<sup>5</sup>

We summarise the structure of our base-line model in the following tables 1 and 2.

Table 1					
	Agent			Bank	
	$\alpha$	$\beta$	$\phi$	$\gamma$	$\delta$
Period 1	Poor in goods/money	Poor in goods/money	Rich in goods/money	Weakly capitalised	Well capitalised
Period 2	Richer in goods	Richer in goods	Poorer in goods		

Table 2		
State $i$	State $ii$	State $iii$
Good state (Arrow Security pays-off)	Bad state	Bad state

We have chosen to begin with this specification since it is the simplest version possible given that we need at least two *heterogenous* banks in order to analyse the *intra-sector* contagion effect within the banking sector via their interaction in the interbank and asset markets and the possible *inter-sector* contagion effect involving the real sector via the credit, deposit, asset and commodity markets. Most importantly, by allowing two separate *defaultable* consumer loan markets, default in one market can produce an additional channel of contagion to the other and to the rest of the economy; a 'consumer loan contagion' channel.

In the following section (2.1) we formally summarise the agents' optimisation problem and the market clearing conditions. Section (2.2) then explains the resulting initial equilibrium given the exogenous parameters. Section (2.3) shows the results of a number of comparative statics exercise.

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<sup>5</sup> An assumption that we shall relax in section 3.

## 2.1 The Agents' Optimisation Problems and Market Clearing Conditions

### 2.1.1 Household $\alpha$ and $\beta$ 's Optimisation Problem

Each consumer  $h \in \{\alpha, \beta\}$  maximises his payoff, which is his utility of consumption minus the (non-pecuniary) default penalty he incurs if he does not pay back his loans. He also observes his cash-in-advance and quantity constraints in each period. These constraints are consistent with the timeline of the model.

As in Goodhart et al. (2003) and Tsomocos (2003a and b), we assume that asset and loan markets clear automatically via a background clearinghouse whereas commodity markets are more sluggish. Put differently, agents cannot use contemporaneous receipts from commodities to engage in other purchases.

$$\max_{\{b_0^h, q_s^h, v_{sb}^h\}, s \in S} \Pi^h = [\chi_0^h - c_0^h(\chi_0^h)^2] + \sum_{s \in S} [\chi_s^h - c_s^h(\chi_s^h)^2] - \sum_{s \in S} \lambda_{sb}^h \max[0, \mu^{h^b} - v_{sb}^h \mu^{h^b}]$$

subject to

$$b_0^h \leq \frac{\mu^{h^b}}{(1 + r^\gamma)} \quad (1)$$

(i.e. expenditure for commodity  $\leq$  borrowed money from the consumer loan market)

$$\chi_0^h \leq \frac{b_0^h}{p_0} \quad (2)$$

(i.e. consumption  $\leq$  amount of goods purchased)

$$v_{sb}^h \mu^{h^b} \leq \Delta(2) + p_s q_s^h + m_s^h, \quad s \in S \quad (3)$$

(i.e. loans repayment  $\leq$  money at hand + receipts from sales of commodity + initial private monetary endowment in state  $s$ )

$$0 \leq e_s^h, \quad s \in S \quad (4)$$

(i.e.  $0 \leq$  endowments of commodities)

$$\chi_s^h \leq e_s^h - q_s^h, \quad s \in S \quad (5)$$

(i.e. consumption  $\leq$  initial endowment - sales)

where,<sup>6</sup>

$\Delta(x) \equiv$  the difference between RHS and LHS of inequality  $(x)$ ,

$b_s^h \equiv$  amount of fiat money spent by  $h \in H$  to trade in the market of commodity,  $s = \{0\} \cup S$ ,

$q_s^h \equiv$  amount of commodity offered for sales by  $h \in H$ ,  $s = \{0\} \cup S$ ,

$\mu^{h^b} \equiv$  amount of fiat money agent  $h^b \in H^b = \{\alpha^\gamma, \beta^\delta\}$  chooses to borrow from his nature selected bank  $b^7$ ,

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<sup>6</sup>Since the notations used in this paper are extensive, we also summarise them in a separated glossary section (section 8).

<sup>7</sup>i.e. Mr.  $\alpha$  borrows from bank  $\gamma$  whereas Mr.  $\beta$  borrows from bank  $\delta$ .

$v_{sb}^h \equiv$  the corresponding rates of repayment in the loan market by household  $h^b$  to his nature-selected bank  $b$  in states  $s \in S$ ,

$\chi_s^h \equiv$  commodity consumption by  $h \in H$  in state  $s \in S$ ,

$\lambda_{sb}^h \equiv$  (non-pecuniary) penalties imposed on  $h$  when contractual obligations in the consumer loan market are broken,

$r^b \equiv$  lending rate offered by bank  $b$ ,

$p_s \equiv$  commodity price in  $s = \{0\} \cup S$ ,

$m_s^h \equiv$  monetary endowment of household  $h$  in states  $s = \{0\} \cup S$ ,

$e_s^h \equiv$  commodity endowment of household  $h$  in states  $s = \{0\} \cup S$ , and

$c_s^l \equiv$  exogenous parameters in the utility/profit functions of agent  $l$  where  $l \in H \cup B$ .

### 2.1.2 Household $\phi$ 's Optimisation Problem

Mr.  $\phi$ 's maximisation problem is as follows:

$$\max_{\{q_0^\phi, b_s^\phi, b_j^\phi, d_\gamma^\phi\}, s \in S} \Pi^\phi = [\chi_0^\phi - c_0^\phi(\chi_0^\phi)^2] + \sum_{s \in S} [\chi_s^\phi - c_s^\phi(\chi_s^\phi)^2]$$

$$b_j^\phi + d_\gamma^\phi \leq m_0^\phi \quad (6)$$

(i.e. expenditures for the Arrow securities + bank deposits  $\leq$  initial private monetary endowments)

$$q_0^\phi \leq e_0^\phi \quad (7)$$

(i.e. sales of commodity  $\leq$  endowments of commodity)

$$\chi_0^\phi \leq e_0^\phi - q_0^\phi \quad (8)$$

(i.e. consumption  $\leq$  initial endowment - sales)

$$b_i^\phi \leq \Delta(6) + p_0 q_0^\phi + d_\gamma^\phi(1 + r^\gamma) + \frac{b_j^\phi}{\theta} \quad (9)$$

(i.e. expenditures for commodity in state  $i \leq$  cash at hand + receipts from sales of commodity from period  $t = 1$  + deposits and interest payment + asset deliveries)

$$b_s^\phi \leq \Delta(6) + p_0 q_0^\phi + d_\gamma^\phi(1 + r^\gamma), \quad s = \{ii, iii\} \quad (10)$$

(i.e. expenditures for commodity in states  $ii$  and  $iii \leq$  cash at hand + receipts from sales of commodity from period  $t = 1$  + deposits and interest payment)

$$\chi_s^\beta \leq \frac{b_s^\phi}{p_s} \quad (11)$$

(i.e. consumption  $\leq$  purchases)

where,

$b_j^\phi \equiv$  amount of money placed by Mr.  $\phi$  in the Arrow security market,

$d_\gamma^\phi \equiv$  amount of money that Mr.  $\phi$  deposits with bank  $\gamma$ , and

$\theta \equiv$  asset price.

### 2.1.3 Bank $\gamma$ 's Optimisation Problem

Bank  $\gamma$  (similarly for bank  $\delta$ ) maximises its profits in  $t = 2$  and suffers a capital requirement violation penalty proportional to its capital requirement violation. Moreover, it observes its liquidity constraints as described in the timeline of the model in figure 1.

Bank  $\gamma$ 's optimisation problem is as follows:

$$\max_{\{\mu^\gamma, \bar{m}^\gamma, q_j^\gamma\}} \Pi^\gamma = \sum_{s \in S} \pi_s^\gamma - \sum_{s \in S} \lambda_{ks}^\gamma \max[0, \bar{k} - k_s^\gamma]$$

subject to

$$\bar{m}^\gamma \leq \frac{\mu^\gamma}{(1 + \rho)} + d_\gamma^\phi + \theta q_j^\gamma \quad (12)$$

(i.e. credit extension  $\leq$  money at hand + interbank loans + consumer deposits + receipt from asset sales)

$$\mu^\gamma + q_j^\gamma + (1 + r^\gamma) d_\gamma^\phi \leq \Delta(12) + v_{i\gamma}^\alpha (1 + r^\gamma) \bar{m}^\gamma + e_i^\gamma \quad (13)$$

(i.e. interbank loan repayment + expenditure for asset deliveries + deposit repayment  $\leq$  money at hand + loan repayment + initial capital endowment in state  $i$ )

$$\mu^\gamma + (1 + r^\gamma) d_\gamma^\phi \leq \Delta(12) + v_{s\gamma}^\alpha (1 + r^\gamma) \bar{m}^\gamma + e_s^\gamma, \quad s = \{ii, iii\} \quad (14)$$

(i.e. interbank loan repayment + deposit repayment  $\leq$  money at hand + loan repayment + initial capital endowment in state  $s = \{ii, iii\}$ )

where,

$\pi_s^\gamma = \Delta(13)$  for  $s = i$ , and  $\Delta(14)$  for  $s = \{ii, iii\}$

$k_s^\gamma = \frac{e_s^\gamma}{\bar{w} v_{s\gamma}^\alpha (1 + r^\gamma) \bar{m}^\gamma}$ ,  $s \in S$ ,

$\bar{k} \equiv$  capital adequacy requirement set by the regulator,

$\lambda_{ks}^b \equiv$  capital requirements' violation penalties on bank  $b \in B$  in state  $s \in S$  set by the regulator,

$\bar{w} \equiv$  risk weight for consumer loans,

$\bar{m}^b \equiv$  amount of credit that bank  $b \in B$  extends,

$q_j^\gamma \equiv$  bank  $\gamma$ 's quantity supply of Arrow securities,

$e_s^b \equiv$  initial capital endowment of bank  $b \in B$  in state  $s = \{0\} \cup S$ ,

$\rho \equiv$  interbank rate, and

$\mu^\gamma \equiv$  amount of money that bank  $\gamma$  borrows from the interbank market.

### 2.1.4 Bank $\delta$ 's Optimisation Problem

Bank  $\delta$ 's optimisation problem is as follows:

$$\max_{\{d^\delta, \bar{m}^\delta, b^\delta\}} \Pi^\delta = \sum_{s \in S} \pi_s^\delta - \sum_{s \in S} \lambda_{ks}^\delta \max[0, \bar{k} - k_s^\delta]$$

subject to

$$d^\delta \leq e_0^\delta \quad (15)$$



(i.e. deposits in the interbank market  $\leq$  initial capital endowment)

$$\bar{m}^\delta + b_j^\delta \leq \Delta(15) \quad (16)$$

(i.e. credit extension + expenditure for asset  $\leq$  money at hand)

$$0 \leq \Delta(16) + \frac{b_j^\delta}{\theta} + v_{i\delta}^\beta \bar{m}^\delta (1 + r^\delta) + d^\delta (1 + \rho) + e_i^\delta \quad (17)$$

(i.e.  $0 \leq$  money at hand + money received from asset payoffs + loan repayments in state  $i$  + interbank deposits and interest payment + initial capital endowment in state  $i$ )

$$0 \leq \Delta(16) + v_{s\delta}^\beta \bar{m}^\delta (1 + r^\delta) + d^\delta (1 + \rho) + e_s^\delta, s = \{ii, iii\} \quad (18)$$

(i.e.  $0 \leq$  money at hand + loan repayments in state  $s = \{ii, iii\}$  + interbank deposits and interest payment + initial capital endowment in state  $s = \{ii, iii\}$ )

where,

$\pi_s^\delta = \Delta(17)$  for  $s = \{i\}$ , and  $\Delta(18)$  for  $s = \{ii, iii\}$ ,

$$k_i^\delta = \frac{e_i^\delta}{\bar{\omega} v_{i\delta}^\beta (1 + r^\delta) \bar{m}^\delta + \omega d^\delta (1 + \rho) + \bar{\omega} \frac{b_j^\delta}{\theta}},$$

$$k_s^\delta = \frac{e_s^\delta}{\bar{\omega} v_{s\delta}^\beta (1 + r^\delta) \bar{m}^\delta + \omega d^\delta (1 + \rho)}, \text{ for } s = \{ii, iii\},$$

$\omega$  ( $\bar{\omega}$ )  $\equiv$  risk weights for interbank market deposits (the Arrow security),

$b_j^\delta \equiv$  amount of money placed by bank  $\delta$  in the market of the Arrow security,

$d^\delta \equiv$  bank  $\delta$ 's interbank deposits, and

$M^{CB} \equiv$  money supply.

### 2.1.5 Market Clearing Conditions

There are 8 markets in the model (one commodity in  $t = 1$  and three in  $t = 2$ , one asset, the interbank and two consumer loan markets). Each of these markets determine a price that equilibrates demand and supply in equilibrium.<sup>8</sup>

$$p_0 = \frac{b_0^\alpha + b_0^\beta}{q_0^\phi} \quad (\text{i.e. commodity market at } t = 1 \text{ clears}) \quad (19)$$

$$p_s = \frac{b_s^\phi}{q_s^\alpha + q_s^\beta}, s \in S \quad (\text{i.e. commodity market at } t = 2, s \in S \text{ clears}) \quad (20-22)$$

$$1 + \rho = \frac{\mu^\gamma}{M^{CB} + d^\delta} \quad (\text{i.e. interbank market clears}) \quad (23)$$

$$1 + r^\gamma = \frac{\mu^{\alpha^\gamma}}{\bar{m}^\gamma} \quad (\text{i.e. bank } \gamma \text{'s loan market clears}) \quad (24)$$

$$1 + r^\delta = \frac{\mu^{\beta^\delta}}{\bar{m}^\delta} \quad (\text{i.e. bank } \delta \text{'s loan market clears}) \quad (25)$$

$$\theta = \frac{b_j^\delta + b_j^\phi}{q_j^\gamma} \quad (\text{i.e. asset market clears}) \quad (26)$$

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<sup>8</sup>The price formation mechanism is identical to the offer-for-sale mechanism in Dubey and Shubik (1978). The denominator of each of the expressions (19-26) represents the supply side whereas the numerator divided by the price corresponds to the demand. Note that this price formation mechanism is well-defined both in, and out of, equilibrium.

### 2.1.6 Equilibrium

Let  $\sigma^h = (b_0^h, q_s^h, v_{sb}^h) \in R \times R^3 \times R^3$  for  $h \in \{\alpha, \beta\}$ ;  $\sigma^\phi = (q_0^\phi, b_s^\phi, b_j^\phi, d_\gamma^\phi) \in R \times R^3 \times R \times R$ ;  $\sigma^\gamma = (\mu^\gamma, \overline{m}^\gamma, q_j^\gamma) \in R^3$ ;  $\sigma^\delta = (d^\delta, \overline{m}^\delta, b_j^\delta) \in R^3$ . Also, let  $\eta = (p_0, p_1, p_2, p_3, \rho, r^\gamma, r^\delta, \theta)$ ,  $B^h(\eta) = \{\sigma^h : (1) - (5) \text{ hold}\}$  for  $h \in \{\alpha, \beta\}$ ,  $B^\phi(\eta) = \{\sigma^\phi : (6) - (11) \text{ hold}\}$ ,  $B^\gamma(\eta) = \{\sigma^\gamma : (12) - (14) \text{ hold}\}$ ,  $B^\delta(\eta) = \{\sigma^\delta : (15) - (18) \text{ hold}\}$ . We say that  $(\sigma^\alpha, \sigma^\beta, \sigma^\phi, \sigma^\gamma, \sigma^\delta; p_0, p_1, p_2, p_3, \rho, r^\gamma, r^\delta, \theta)$  is a monetary equilibrium with commercial banks and default iff:

- (i) (a)  $\sigma^h \in \underset{\sigma^h \in B^h(\eta)}{\text{Argmax}} \Pi^h(\chi^h)$ ,  $h \in \{\alpha, \beta, \phi\}$
- (b)  $\sigma^b \in \underset{\sigma^b \in B^b(\eta)}{\text{Argmax}} \Pi^b(\pi^b)$ ,  $b \in \{\gamma, \delta\}$

and

- (ii) All markets (19)-(26) clear.

## 2.2 Exogenous Parameters and Initial Equilibrium

The values of the exogenous variables are summarised in table I of appendix I. The numbers chosen are mostly illustrative at this stage; at a later stage in this research we hope to calibrate a revised version of the paper against real data. Thus, of itself a simulation of this kind is not particularly interesting, though it was, because of the size of the system, technically quite difficult. However, of greater interest are the comparative statics arising from varying the chosen inputs to the system. Armed with the propositions of the general model, we can trace the equilibria of the simulations and study how the multiple markets and choice variables interact. In turn, we can see how the many system-wide effects determine price, interest rates and allocations.

The values of commodity and monetary endowments of households are chosen so that Mr.  $\alpha$  and  $\beta$  (Mr.  $\phi$ ) are poor (rich) at  $t = 1$ , and therefore are net borrowers (lender). Similarly, the selected value of capital endowments of banks ensures that bank  $\gamma$  is relatively poor at  $t = 1$  and has to borrow from the interbank and asset markets, and vice versa for bank  $\delta$ . Furthermore, the value of regulatory capital adequacy requirement is chosen to be sufficiently high (0.4) in order to ensure that all banks violate their capital requirements and thus are penalised accordingly. The risk weight for consumer loans is set to 1, while that of interbank loans and assets are set to 0.5, to reflect the fact that loans are defaultable and therefore riskier than the other two types of assets. The rest of the exogenous variables/parameters are chosen to ensure a reasonable initial Monetary Equilibrium with Commercial Banks and Default (MECBD). The values of the initial equilibrium are shown in table II of appendix I. In particular, they are chosen to ensure that the values of all the repayment rates are realistic, and the interbank interest rate is lower than both the interest rates charged by both banks since interbank loans are assumed to be default free and thus do *not* include a default premium. Finally, the loan rate of bank  $\gamma$  is higher than that of bank  $\delta$  so that Mr.  $\phi$  chooses bank  $\gamma$  to deposit.

## 2.3 Results

This section shows the effects of changes in the exogenous variables/parameters of the model. Table III of appendix I describes the directional effects on endogenous variables of changing various parameters listed in the first column. We solve the model using *Mathematica*. We first guessed the initial equilibrium described in table II of appendix I. Then using Newton's method,

we calculated numerically how the initial equilibrium changes as we vary each parameter at a time.

The analysis is conducted using the principles derived in Goodhart et al. (2003). Besides the non-neutrality of both regulatory and monetary policies, we have also established the following results:

(i) Liquidity Structure of Interest Rates:

Since base money is fiat and the horizon is finite, in the end no household will be left with fiat money. Thus, all households will finance their loan repayments to commercial banks via their private monetary endowment and the initial capital endowments of banks (recall that banks' profit is distributed to their shareholders). However, since we allow for defaults, the total amount of interest rate repayments is adjusted by the corresponding anticipated default rates. In sum, aggregate *ex post* interest rate payments adjusted for default to commercial banks is equal to the total amount of outside money (i.e. sum of private monetary and initial commercial banks' endowments). In this way, the overall liquidity of the economy and endogenous default co-determine the structure of interest rates.

(ii) Quantity Theory of Money Proposition:

The model possesses a non-mechanical quantity theory of money. Velocity will always be less than or equal to one (one if all interest rates are positive). However, since quantities supplied in the markets are chosen by agents (unlike the representative agent model's *sell-all* assumption), the real velocity of money, that is how many real transactions can be moved by money per unit of time, is endogenous. The upshot of the analysis is that nominal changes (i.e. changes in monetary policy) affect both prices and quantities.

(iii) Fisher Effect:

The nominal interest rate is equal to the real interest rate plus the expected rate of inflation.

We conclude this section by highlighting the key results that we obtain from this numerical exercise.

### 2.3.1 An Increase in Money Supply

Let the Central Bank engage in expansionary monetary policy by increasing the money supply ( $M^{CB}$ ) in the interbank market (or equivalently lowering the interbank interest rate( $\rho$ )) (see row 1 of table III in appendix I). Lowering the interbank rate induces bank  $\gamma$  to borrow more from the interbank market and therefore to increase its supply of loans to Mr.  $\alpha$ , pushing down the corresponding lending rate  $r^\gamma$ . Consequently, agent  $\phi$  reduces his deposits in bank  $\gamma$  and switches his investment to the asset market, pushing the asset price up slightly. Given lower expected rates of return from investing in the interbank and asset markets, bank  $\delta$  invests less in these markets and switches to supply more loans to Mr.  $\beta$ , causing the corresponding lending rate  $r^\delta$  to decline.

Since more money chases the same amount of goods, by the quantity theory of money proposition, prices in both periods and all states increase. Prices in state  $i$  increase the most, since Mr.  $\phi$  has increased his demand for Arrow securities and therefore has more income to spend on commodities in state  $i$ . Lower interest rates make trade more efficient, since the increase in liquidity results in lower default rates for both Mr.  $\alpha$  and Mr.  $\beta$ , especially in state  $i$  where Arrow securities pay off.<sup>9</sup> Thus aggregate consumer default falls.

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<sup>9</sup>Since our model is transaction based, lower interest rates generate lower 'transaction' costs to agents who borrows. In principle, default therefore falls. In the limit, when interest rates are equal to zero and markets are complete, full pareto optimality is obtained (see Corollary 2 of Tsomocos (2003a) for further discussion).

Turning now to capital requirements' violation, both banks break their capital requirement constraints more than before, particularly bank  $\delta$ . Higher repayment rates and credit extension over-compensate for the decrease in interest rates and thus, for given capital, risk weighted total assets increase. Bank  $\delta$ , which is relatively richer than bank  $\gamma$ , violates its requirements even more, since the marginal benefit of the increased profits is greater than the marginal cost of the capital requirement violation. Thus, given an initially adverse capital requirement position (and also banks' inability to access capital markets to raise new equity), expansionary monetary policy worsens their capital adequacy condition. The reason is that the extra profit effect dominates the capital requirement violation cost.

Both regulatory and monetary policies affect credit extension. In addition, default and capital requirements' violation have different marginal costs (due to the different penalties). So, there exists a trade-off between earning a greater excess return through interest receipts and the cost of capital requirements' violation. Thus, the interaction of the capital adequacy ratio and credit extension should be analysed contemporaneously in order to determine the optimal composition of banks' assets. We also note that lower defaults on consumer borrowing does not necessarily improve capital assets' ratios since profit-maximising banks will respond by lending even more.

As far as the welfare of the agents is concerned, the utility of Mr.  $\alpha$  and the profit of bank  $\gamma$  improve whereas profits of bank  $\delta$  deteriorate. The welfare of Mr.  $\beta$  and Mr.  $\phi$  remains almost unaffected (slight improvement). The welfare improvement of Mr.  $\alpha$  results from lower interest rates, (and consequently a higher repayment rate on his loans and thus lower default penalties). The higher expected prices in period 2 also contribute to the higher repayment rates, since higher prices imply higher expected income from selling commodities. Thus, as predicted by the Fisher effect, higher prices imply lower real interest rates at  $t = 1$  since nominal interest rates fall. The profitability of bank  $\gamma$  increases, mainly due to lower consumer default which dominates the higher cost of capital requirements' violations. However, the positive spillover effect of lower consumer default for bank  $\delta$  fails to dominate the lower revenue, due to lower interest rates, whose profitability therefore decreases along with higher capital requirements violations.

In sum, even though expansionary monetary policy improves aggregate consumer default rates, it does not necessarily induce less financial fragility. Higher liquidity provides an incentive for profit-maximising commercial banks to expand without necessarily improving their capital requirements condition.

### 2.3.2 An Increase in the Loan Risk Weights applied to Capital Requirements

An assessment of the effect of an increase in the risk weights on loans for both banks ( $\bar{\omega}$ ) (see row 3 of table III in appendix I) underscores the argument that those agents who have more investment opportunities, and therefore greater flexibility, can mitigate the effect of a negative shock by restructuring their portfolios. In this simulation the initial condition of the economy is adverse in the sense that capital requirements are binding and there is no access to the capital markets to raise new equity; so the impact is procyclical. Bank  $\delta$  will further reduce credit extension to avoid the extra cost of the additional capital requirements' violation penalty, and bank  $\gamma$  in particular will increase its violation since it cannot switch its investments to maintain its profitability. Consequently, its payoff will be severely affected both from reduced interest rate receipts and also the higher penalties for capital requirements' violation. In contrast, bank  $\delta$  reduces investments in both the loan and interbank markets and increases its investment in the asset market.

Bank  $\gamma$ , anticipating the higher expected capital requirement violation penalty, will increase its credit extension to lessen its profit reduction, by borrowing more from the asset market and thus lowering the asset price. Since bank  $\gamma$  will charge lower interest rates in order to increase credit extension, deposits from Mr.  $\phi$  decrease and, given lower asset prices, he switches to invest more in the asset market. In contrast, bank  $\delta$ , which diversifies away from the loan market, increases its interest rates. Moreover, reduced investments in the interbank market by bank  $\delta$  increase the interbank market interest rate. Tighter credit reduces commodity prices in all periods, except state  $i$  where the Arrow security pays and there is extra liquidity in the economy. Higher interbank rates imply higher default rates except in the case where the Arrow security pays off (i.e. state  $i$ ). So, default by both agents increases on average, (even though both of them maintain higher repayment rates in state  $i$ ), because of tighter credit market conditions for Mr.  $\beta$  and lower expected income for both Mr.  $\alpha$  and Mr.  $\beta$ .

The profitability for bank  $\gamma$  is reduced substantially, whereas bank  $\delta$ 's ability to restructure its portfolio generates slightly positive profits, even though the aggregate profit of the banking industry is reduced. Paradoxically, though, Mr.  $\alpha$ 's welfare is improved. Because in effect bank  $\gamma$  follows a countercyclical policy in response to the higher risk weights, so lower interest rates help Mr.  $\alpha$  to borrow more cheaply and increase his consumption in period 1, thus slightly improving his utility. However, Mr.  $\beta$  is hurt by the higher interest rate charged by bank  $\delta$ . Finally, Mr.  $\phi$ 's utility is almost unchanged (with ambiguous sign), since the lower purchasing power resulting from lower bank deposit rates is more than offset by a higher return on his asset investment.

Regulatory policy may be seen as a mirror image of monetary policy, since it directly affects credit extension via the capital requirements' constraint. Moreover, banks without well-diversified portfolios, and thus not so many investment opportunities, follow a countercyclical credit extension policy that hurts them, but benefits their respective clients. The countercyclical credit extension policy of not-well-diversified banks may also be thought of as a built-in-stabilizer in the economy when regulatory policy becomes tighter and the economy faces a danger of multiplicative credit contraction. On the contrary, banks that can quickly restructure their portfolios transfer the negative externalities of higher risk weights to their clients. Thus, restrictive regulatory policy in periods of economic adversity may enhance financial fragility by inducing lower profitability, higher default and further capital requirement violations.<sup>10</sup>

### 2.3.3 Summary of the Base-line Model Results

All the results of the various comparative statics are tabulated in table III of appendix I. Their interpretation and analysis can be undertaken using the principles we have used so far. Here we recapitulate the key results obtained from these comparative statics. First, in an economic environment in which capital constraints are binding, which may be viewed as representing adverse economic conditions, expansionary monetary policy can aggravate financial fragility since the extra liquidity injected by the Central Bank may be used by certain banks to expand, and in some senses to 'gamble for resurrection', worsening their capital position, and therefore the overall financial stability of the economy. Thus, a trade-off between efficiency and financial

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<sup>10</sup>As shown in Catarineu-Rabell, Jackson and Tsomocos (2003), if banks are allowed to choose the risk-weights of their assets, they would opt for countercyclical risk-weight setting. In this way, they would lessen the profit reduction induced by falling loan opportunities in the economy. And if they are not allowed to do so by the regulatory authorities, then they would choose procyclical weights rather than forward looking ones, thus exacerbating credit contraction in the economy.

stability need not exist only for regulatory policies, but also for monetary policy.

Second, agents which have more investment opportunities can deal with negative shocks more effectively by using their flexibility to restructure their investment portfolios quickly as a means of transferring ‘negative externalities’ to other agents with a more restricted set of investment opportunities. This result has various implications. Among others, banks which have no well-diversified portfolios tend to follow a countercyclical credit extension policy in face of a negative regulatory shock in the loan market (e.g. tighter loan risk weights). In contrast, banks which can quickly restructure their portfolio tend to reallocate their portfolio away from the loan market, thus following a procyclical credit extension policy. Moreover, regulatory policies which are selectively targeted at different groups of banks can produce very *non-symmetric* results, e.g. an increase in capital requirement penalty of bank  $\gamma$  vs. bank  $\delta$  (see rows 6 and 7 of table III in appendix I). When the policy is aimed at banks which have more investment opportunities, e.g. bank  $\delta$ , much less contagion to the rest of the economy occurs since those banks simply restructure their portfolios between interbank and asset markets without greatly perturbing the credit market, thus not affecting substantially interest rates and prices in the economy. On the contrary, when the same policy is targeted at banks which have relatively limited investment opportunities, e.g. bank  $\gamma$ , they are forced to ‘bite the bullet’ by altering their credit extension. This produces changes in a series of interest rates, and therefore the cost of borrowing for agents. This in turn produces a contagion effect to the *real* sector in the economy.

Thirdly, an improvement such as a positive productivity shock, which is concentrated in one part of the economy, does not necessarily improve overall welfare and profitability of the economy. The key reason for this lies in the fact that our model has *heterogenous* agents and therefore possesses various feedback channels which are all active in equilibrium. Thus, a positive shock in one specific sector can produce a negative contagion effect in others, even possibly causing the welfare and/or profitability of the whole economy to fall. For example, if the commodity endowment of household  $\alpha$  increases in state  $i$  (see row 9 of table III in appendix I), his increased revenue leads him to increase his repayment rate on his loans. This in turn pushes bank  $\gamma$ ’s lending rate down considerably. This results in lower profitability for bank  $\gamma$ , because higher repayments are outweighed by lower interest rate payments. Moreover, the fall of commodity prices also adversely affects Mr.  $\beta$  whose income from commodity sales in state  $i$  drops.

### 3 Extension: Endogenous Defaults in the Interbank and Deposit Markets

The comparative statics results shown in the previous section can be varied to incorporate a different set of assets. So, we next, briefly, describe an extended version of the base-line model. In addition to our attempt to examine the robustness of our results in the previous section, this extension aims at illuminating how the effect of various shocks can generate contagion effects via the interbank and deposit markets. To that end, we modify the structure of the model given in section 2.

First, we allow endogenous defaults in the interbank market, i.e. bank  $\gamma$  can default on its interbank loans. Second, we allow separated deposit markets.<sup>11</sup> Moreover, Mr.  $\phi$  has a choice to deposit his money with either bank. Bank  $\gamma$ ’s deposit rate differs from that of bank  $\delta$  since

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<sup>11</sup>Recall that in the previous comparative static we assume perfect financial intermediation, i.e. a perfectly elastic demand for deposits by bank  $\gamma$  at the rate of interest equal to its lending rate.

it is allowed to default on its deposit obligation to Mr.  $\phi$ . Bank  $\gamma$  may also default on its loans from the interbank market. Third, in order to incorporate these additional complexities while retaining the model tractability, we simplify the model by removing the Arrow security. Finally, we assume that the cost of default in the interbank and deposit market is quadratic. This in turn implies that the marginal cost of default in these markets is greater as the size of borrowing is larger. The detailed optimisation problems are given in appendix II. Moreover, tables I and II of appendix III summarise the values of exogenous parameters and the resulting initial equilibrium.

### 3.1 Results

Table III of appendix III describes the directional effects on endogenous variables of increasing various parameters listed in the first column.

#### 3.1.1 An Increase in Money Supply

As in section 2.3.1, let the Central Bank engage in expansionary monetary policy by increasing the money supply ( $M^{CB}$ ) in the interbank market (or equivalently lowering the interbank market rate ( $\rho$ )) (see row 1 of table III in appendix III). Given a lower rate of return on interbank market investment, bank  $\delta$  borrows less from the deposit market and switches to invest more in the consumer loan market by supplying more credit to Mr.  $\beta$ . Thus bank  $\delta$ 's deposit and lending rates both decrease. As the deposit rate of bank  $\delta$  falls, Mr.  $\phi$ , who now has the option to diversify his deposits between banks  $\gamma$  and  $\delta$ , deposits more with bank  $\gamma$ , causing its deposit rate to decline as well. Moreover, given a lower cost of borrowing in the deposit and interbank markets, bank  $\gamma$  borrows more from these markets and increases its credit extension, thus lowering its lending rate offered to Mr.  $\alpha$ .

Due to the fact that bank  $\gamma$  borrows more both from the interbank and deposit markets and the default penalty is now quadratic, it increases its repayment rates in these markets. Given increased liquidity in the economy, all prices increase in both periods, however more in the first period when monetary policy loosens.<sup>12</sup> This in turn generates more income to households; including those who sell their commodities in the second period. Thus, they all increase their repayment rates in the consumer loan market. Bank  $\gamma$  violates more capital requirements because its risk-weighted assets increase and it does not have access to equity markets. Their risk-weighted assets increase because the effects of higher credit extension and higher borrowers' repayment rates dominate the effect of lower lending rates. In contrast, bank  $\delta$  violates less capital requirements since the effect of lower lending rates coupled with the effect of lower interbank market investment dominate the effects of higher credit extension and higher borrowers' repayment rates.

As far as welfare is concerned, both borrowers, namely Mr.  $\alpha$  and  $\beta$ , improve their payoffs due to lower borrowing cost and lower default penalties since they increase their repayment. However, the creditor who in our case is Mr.  $\phi$  suffers from lower deposit rates, thus his expected income falls. This causes him to reduce his consumption in period 2. Similarly, both banks end up with a lower payoff. This is because the negative effect of lower lending rates dominates the positive effect of higher repayment rates by both Mr.  $\alpha$  and  $\beta$ .

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<sup>12</sup>Note that in most models with liquidity constraints, there is always an overshooting phenomenon in the period when a policy change occurs. For the same phenomenon in an international context, see Geanakoplos and Tsomocos (2002).

In sum, since there are separate deposit markets and default in the deposit market is also allowed, the effects we observed in the previous comparative static are now accentuated in the banking sector where the profitability of both banks is reduced and financial fragility is further increased. On the other hand, the welfare of both borrowers is now slightly increased due to the presence of separate deposit and lending markets.

### 3.1.2 An Increase in the Loan Risk Weights applied to Capital Requirements

A tightening of regulatory policy by increasing risk weights on loans of both banks (see row 3 of table III in appendix III) will have similar effects as in section 2.3.2. However, some differences will be noticeable particularly in the banking sector because we now allow for default in both the interbank and deposit markets, and deposit and lending markets are now separated (i.e. no perfect financial intermediation).

As before, tighter regulatory policy is a mirror image of contractionary monetary policy, and so the interbank rate increases. Bank  $\delta$  will further reduce its credit extension to avoid capital requirements' violation penalties, whereas bank  $\gamma$  whose portfolio is limited increases its credit extension to maintain its profitability. So, bank  $\gamma$ 's lending rate decreases and bank  $\delta$ 's increases. However, bank  $\delta$  has less flexibility than before because interbank loans are now defaultable and the deposit/lending spread is variable. In other words, both bank  $\gamma$  and the depositor can adjust their behaviour in the light of bank  $\gamma$ 's action.

We introduce quadratic default penalties that imply that the marginal cost of defaulting is increasing. Thus, as regulation tightens, bank  $\gamma$  not only reduces its borrowing from the interbank market, but also lowers its repayment rate to support its profitability. Bank  $\delta$  rationally expects higher defaults, and thus lowers its deposits in the interbank market, pushing the interbank rate even higher. Given higher interbank rates, bank  $\gamma$  increases its deposit demand offering higher deposit rate to Mr.  $\phi$  who in turn deposits more with bank  $\gamma$  and less with bank  $\delta$ . This pushes up the deposit rate of bank  $\delta$ . Finally, since bank  $\gamma$  increases its deposit demand and reduces its interbank loans, it increases its repayment of deposits while reduces its repayment of interbank borrowing, given quadratic default penalties.<sup>13</sup>

Since both deposit rates increase, Mr.  $\phi$  receives more income from his investments. He is the buyer of commodities in period 2 and since more money chases the same quantity of goods, by the quantity theory of money proposition, prices increase in the second period. Note that this is in contrast with what happened in the previous comparative static where there was no separated deposit market, which in turn implied that deposit and lending rates were, by definition, restricted to be the same, since tighter credit was automatically translated to lower income to depositors as well. Here we face a wealth redistribution from the banks to their depositors.

Mr.  $\alpha$  and  $\beta$ , anticipating higher expected income from their commodity sales, increase their repayment rates on their respective loans. Finally, both banks, bank  $\gamma$  in particular, increase their capital requirements' violation. Again the bank with the richer portfolio will follow a procyclical credit extension policy, whereas the one with the more restricted portfolio will follow a countercyclical policy.

Turning to the welfare of the economy. Mr.  $\alpha$ 's welfare is improved as before. However, unlike previously, Mr.  $\beta$ 's welfare remains unaffected since higher prices in the second period allow him to pay back his loans without increasing his commodity sales. Similarly, Mr.  $\phi$ 's welfare remains unaltered since the positive effect from higher deposit interest payments is offset

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<sup>13</sup>Endogenous default and the ensuing penalties can be seen as altering the effective payoff of banks' liabilities which therefore forms an optimal liability portfolio, given its risk preferences.



by the negative effect from higher commodity prices in the second period. As before, bank  $\gamma$  is hurt by higher capital requirements' violation penalties. The main difference, however, lies in the reduced profitability of bank  $\delta$ . This occurs because bank  $\gamma$  now has a default option and the separate deposit markets allow more room for Mr.  $\phi$  to diversify his deposits. Thus, we see that what matters is the number of financial instruments available to an agent relative to others. In other words, when a wide array of instruments such as the default option and separate deposit markets are available to everybody, then banks with stronger and more diversified portfolios cannot simply transfer the negative impact of shocks to the rest of the economy. Indeed, they must bear some of it themselves.

To summarise, as regulatory policy tightens in times of adverse economic conditions, bank profitability is further affected. In addition, default may also increase in the interbank market, thus increasing financial fragility in the economy.

### 3.1.3 Summary of the Extended Model Results

The rest of the results are tabulated in table III of appendix III and can be analysed along the same lines. In principle, they reinforce the conclusion reached in section 2. Expansionary monetary policy may enhance financial fragility in the short run, and banks with more investment opportunities can cope with negative shocks more effectively, thus limiting their profit losses.

When the commodity endowment of Mr.  $\alpha$  increases in state  $i$  (see row 9 of table III in appendix III), bank  $\delta$  supplies less credit to Mr.  $\beta$  and switches to increase its investment in the interbank market. This is so because Mr.  $\beta$  is adversely affected in state  $i$  by lower commodity prices and thus defaults more to bank  $\delta$ . Meanwhile, bank  $\gamma$  has a lower cost of borrowing, and does not decrease its deposit rate commensurately. Indeed, the presence of the deposit markets provides an extra degree of freedom to banks to vary optimally the deposit and lending spread and thus depositors can diversify their deposits. Put differently, this is testimony that a wider array of financial markets, typically, improves economic welfare. Thus, unlike previously, although this shock which directly improves the welfare of one agent may worsen that of the others, aggregate welfare now improves.

Regulatory policies targeted at the relatively more flexible bank  $\delta$ , e.g. an increase in capital requirement penalty of bank  $\delta$  (see row 7 of table III in appendix III), now have more real effects in the economy. This is so because bank  $\delta$  does not any longer have the opportunity to invest in the asset market and consequently changes more forcefully its credit extension policy. Credit extension changes have more direct effects on the real economy since credit multipliers are typically greater than asset multipliers. Put differently, given our initial condition, changes in credit extension work through the budget constraints of agents who, in turn, decide how to spend their extra liquidity. However, changes in the asset investment portfolio of banks affects not only the liquidity of the suppliers (i.e. agents), but also generates a price effect. Thus, the real effect of asset portfolio changes is mitigated as contrasted to the credit extension changes.

The contagion effects of a positive shock now depend largely on where the shock is initiated. Financial fragility in the interbank and deposit markets now depends on the agent who was first affected by the shock. For example, when we conduct money financed fiscal transfer (i.e. an increase in an agent's money endowment) or a productivity shock (i.e. an increase in an agent's commodity endowment) to Mr.  $\alpha$  in state  $i$  (see rows 2 and 9 of table III in appendix III, respectively), average default in the interbank and deposit markets falls. This is so because bank  $\gamma$ , whose client is Mr.  $\alpha$ , borrows from the interbank market. However, the opposite is true when the shocks emanate from Mr.  $\beta$  since his nature-selected bank (bank  $\delta$ ) is a net

lender in the interbank market (see rows 10 and 18 of table III in appendix III).

In conclusion, policies must be context specific since one size does *not* fit all objectives in heterogeneous models. In particular, real business cycle models that rely heavily on the representative agent hypothesis are not able to address policy effects in multi-agent economies. As most of our experiments make clear, contagion and its impact to the various sectors of the economy depends on the origin of the shock.

## 4 Conclusion

Large, and non-linear, models, such as Goodhart, et al (2003), normally do not have closed-form solutions. They have to be solved numerically. This paper provides numerical simulations of simplified versions of the above more general model.

The ability to do this shows that, in some senses, the model ‘works’. Moreover, it can be made to ‘work’ in a massively wide variety of initial starting conditions, e.g. depending on which asset markets are included in each variant of the model, and of comparative static exercises to be run. Indeed, the exercises and results reported in Sections 2.3 and 3 are a hugely boiled-down version, a precis, of the full set of exercises, both those that we have done, and, even more so, those that we, in principle, could do. We selected a small sub-set of starting conditions, and of comparative static exercises, with the aim of being both, (relatively), simple and illustrative.

What then have we illustrated? These insights fall into two general categories. First there are those characteristics of a monetary model which not only hold here, but should hold in any well-organised model. We have emphasised three. The first is what we have termed the ‘quantity theory of money’, whereby monetary changes feed through into price and quantity changes, both in the current and future period ( $t = 1, 2$ ). We have assumed an endowment economy, so the volume of goods is, by definition, fixed. But more, or less, everything else ‘real’ in the system does change, distributions between agents, ‘real’ interest rates, bank profitability, default penalties, etc., etc. The system (and the ‘real world’) is non-neutral.

As noted, our model allows for non-zero expectations of future price inflation. Our model also incorporates the Fisher effect, whereby nominal rate (at  $t = 2$ ) are a function of ‘real’ rates and inflation expectations. Finally ‘real’ rates, and *rate differentials*, are a function of the temporal, and distributional, pattern of endowments (time preference), liquidity (i.e. the amount of money injected into the system), and default risk, (the greater the risk, the higher the required rate).

The second set of insights relates to the implications of the main innovative feature of our model, which is that the real world is heterogeneous; agents and banks are not all alike. This has some, fairly obvious, implications. The result of a shock may depend on the particular agent, part of the economy, on which it falls. The response of a bank to a regulatory change will generally depend sensitively on the particular context in which that bank finds itself, and will vary as that context changes. The result of a shock can often shift the distribution of income, and welfare, between agents in a complex way, which is hard to predict in advance.

In short, heterogeneity leads to greater complexity. What we lose, by including it in our model, is simplicity; what we hope to gain is greater reality. In this latter respect, however, simulations, such as these, are always somewhat lacking. We have chosen the initial conditions, and so the outcome is the somewhat artificial construct of our own assumed inputs.

We accept this, and we offer this paper, and these results, as a stepping-stone, a stop on the route, of our continuing research. The next step will be to take our model, adjusted as

may be necessary, to the actual data, to calibrate inter-actions between existing banks and (sets of) agents. But that, for the time-being, is for the future.

## 5 Appendix I

Table I: Exogenous variables						
Coefficient of risk aversion	Endowment			Penalty		Others
	Commodities	Money	Capital	Default	CAR violation	
$c_0^\alpha = 0.0319$	$e_0^\alpha = 0$	$m_0^\alpha = 0$	$e_0^\gamma = 0.1$	$\lambda_{1\gamma}^\alpha = 0.2$	$\lambda_{1k}^\gamma = 5$	$M^{CB} = 0.193$
$c_1^\alpha = 0.0458$	$e_1^\alpha = 18$	$m_1^\alpha = 0.028$	$e_1^\gamma = 2$	$\lambda_{2\gamma}^\alpha = 0.1$	$\lambda_{2k}^\gamma = 4$	$\bar{\omega} = 1$
$c_2^\alpha = 0.048$	$e_2^\alpha = 18$	$m_2^\alpha = 0.0071$	$e_2^\gamma = 2$	$\lambda_{3\gamma}^\alpha = 0.05$	$\lambda_{3k}^\gamma = 3.7064$	$\omega = 0.5$
$c_3^\alpha = 0.0453$	$e_3^\alpha = 18$	$m_3^\alpha = 0.0373$	$e_3^\gamma = 2$	$\lambda_{1\delta}^\beta = 0.2$	$\lambda_{1k}^\delta = 2.4856$	$\tilde{\omega} = 0.5$
$c_0^\beta = 0.0264$	$e_0^\beta = 0$	$m_0^\beta = 0$	$e_0^\delta = 16$	$\lambda_{2\delta}^\beta = 0.1$	$\lambda_{2k}^\delta = 0.9318$	$A_j = [1 \ 0 \ 0]'$
$c_1^\beta = 0.0371$	$e_1^\beta = 20$	$m_1^\beta = 1.65$	$e_1^\delta = 1.5$	$\lambda_{3\delta}^\beta = 0.05$	$\lambda_{3k}^\delta = 0.9318$	$\bar{k} = 0.4$
$c_2^\beta = 0.0374$	$e_2^\beta = 20$	$m_2^\beta = 2.3679$	$e_2^\delta = 1.5$			
$c_3^\beta = 0.0367$	$e_3^\beta = 20$	$m_3^\beta = 2.1468$	$e_3^\delta = 1.5$			
$c_0^\phi = 0.0347$	$e_0^\phi = 25$	$m_0^\phi = 1.7066$				
$c_1^\phi = 0.0205$	$e_1^\phi = 0$	$m_1^\phi = 0$				
$c_2^\phi = 0.019$	$e_2^\phi = 0$	$m_2^\phi = 0$				
$c_3^\phi = 0.0327$	$e_3^\phi = 0$	$m_3^\phi = 0$				

Table II: Initial Equilibrium					
Prices	Loans/deposits	Capital/Asset ratio	Repayment rate	Commodities	Assets
$p_0 = 1$	$\bar{m}^\gamma = 8.01$	$k_1^\gamma = 0.19$	$v_{1\gamma}^\alpha = 0.94$	$b_0^\alpha = 8.01$	$b_j^\phi = 0.26$
$p_1 = 1.1$	$\bar{m}^\delta = 10$	$k_2^\gamma = 0.193$	$v_{2\gamma}^\alpha = 0.927$	$q_1^\alpha = 9.5$	$b_j^\delta = 4.45$
$p_2 = 1.18$	$d_\gamma^\phi = 1.45$	$k_3^\gamma = 0.194$	$v_{3\gamma}^\alpha = 0.923$	$q_2^\alpha = 8.81$	$q_j^\gamma = 16.9$
$p_3 = 1.34$	$d^\delta = 1.56$	$k_1^\delta = 0.085$	$v_{1\delta}^\beta = 0.9$	$q_3^\alpha = 7.7$	
	$\mu^{\alpha\gamma} = 11.19$	$k_2^\delta = 0.102$	$v_{2\delta}^\beta = 0.89$	$b_0^\beta = 10$	
Interest rates/ Asset price	$\mu^{\beta\delta} = 13.49$	$k_3^\delta = 0.103$	$v_{3\delta}^\beta = 0.88$	$q_1^\beta = 9.5$	
	$\mu^\gamma = 2.1$			$q_2^\beta = 8.2$	
$r^\gamma = 0.4$				$q_3^\beta = 7.29$	
$r^\delta = 0.35$				$q_0^\phi = 18.02$	
$\rho = 0.197$				$b_1^\phi = 20.9$	
$\theta = 0.278$				$b_2^\phi = 20.02$	
				$b_3^\phi = 20.02$	

Table III: Directional Effects of an increase in exogenous parameters on endogenous variables															
	$p_0$	$p_1$	$p_2$	$p_3$	$r^\gamma$	$r^\delta$	$\rho$	$\theta$	$\overline{m}^\gamma$	$\overline{m}^\delta$	$d_\gamma^\phi$	$d^\delta$	$\mu^{\alpha\gamma}$	$\mu^{\beta\delta}$	$\mu^\gamma$
$M$	$\approx$	$\approx$	$\approx$	$\approx$	$-$	$-$	$\approx$	$\approx$	$\approx$	$\approx$	$-$	$\approx$	$\approx$	$\approx$	$\approx$
$m_1^\alpha$	$\approx$	$\approx$	$\approx$	$\approx$	$-$	$+$	$+$	$\approx$	$\approx$	$\approx$	$-$	$+$	$\approx$	$\approx$	$+$
$\overline{w}$	$\approx$	$+$	$-$	$-$	$-$	$+$	$+$	$-$	$+$	$-$	$-$	$-$	$\approx$	$\approx$	$-$
$\lambda_{1\gamma}^\alpha$	$\approx$	$\approx$	$-$	$-$	$-$	$+$	$-$	$\approx$	$\approx$	$\approx$	$-$	$+$	$-$	$\approx$	$+$
$\lambda_{1\delta}^\beta$	$\approx$	$-$	$+$	$+$	$+$	$-$	$+$	$\approx$	$\approx$	$\approx$	$+$	$-$	$\approx$	$-$	$-$
$\lambda_{1k}^\gamma$	$\approx$	$-$	$+$	$+$	$+$	$-$	$-$	$\approx$	$\approx$	$\approx$	$+$	$-$	$\approx$	$\approx$	$-$
$\lambda_{1k}^\delta$	$\approx$	$\approx$	$\approx$	$\approx$	$\approx$	$\approx$	$\approx$	$\approx$	$\approx$	$\approx$	$\approx$	$-$	$\approx$	$\approx$	$-$
$e_0^\phi$	$-$	$+$	$-$	$-$	$-$	$-$	$-$	$\approx$	$\approx$	$\approx$	$-$	$+$	$-$	$-$	$+$
$e_1^\alpha$	$+$	$-$	$+$	$+$	$-$	$+$	$+$	$\approx$	$+$	$-$	$+$	$-$	$\approx$	$\approx$	$-$
$e_1^\beta$	$\approx$	$-$	$+$	$+$	$+$	$-$	$+$	$-$	$-$	$+$	$+$	$-$	$\approx$	$\approx$	$-$
$e_1^\gamma$	$\approx$	$-$	$+$	$+$	$+$	$-$	$-$	$\approx$	$-$	$+$	$+$	$-$	$\approx$	$\approx$	$-$
$e_1^\delta$	$\approx$	$\approx$	$\approx$	$\approx$	$\approx$	$\approx$	$\approx$	$\approx$	$\approx$	$\approx$	$\approx$	$-$	$\approx$	$\approx$	$-$
$c_1^\alpha$	$+$	$-$	$+$	$+$	$-$	$+$	$+$	$\approx$	$+$	$-$	$+$	$-$	$\approx$	$\approx$	$-$
Note: $+$ ( $-$ ) : substantial increase (decrease)															
$\approx$ ( $\approx$ ) :weak increase (decrease), $\approx$ : approximately equal															
$+/-$ :ambiguous effect															

Table III (continue)																		
	$k_1^\gamma$	$k_2^\gamma$	$k_3^\gamma$	$k^\gamma$	$k_1^\delta$	$k_2^\delta$	$k_3^\delta$	$k^\delta$	$k$	$v_{1\gamma}^\alpha$	$v_{2\gamma}^\alpha$	$v_{3\gamma}^\alpha$	$v_\gamma^\alpha$	$v_{1\delta}^\beta$	$v_{2\delta}^\beta$	$v_{3\delta}^\beta$	$v_\delta^\beta$	$v$
$M$	$-$	$\approx$	$\approx$	$\approx$	$\approx$	$\approx$	$\approx$	$\approx$	$\approx$	$+$	$\approx$	$\approx$	$\approx$	$+$	$\approx$	$\approx$	$\approx$	$\approx$
$m_1^\alpha$	$-$	$\approx$	$\approx$	$\approx$	$\approx$	$\approx$	$\approx$	$\approx$	$\approx$	$+$	$\approx$	$\approx$	$\approx$	$+$	$\approx$	$\approx$	$\approx$	$\approx$
$\overline{w}$	$-$	$-$	$-$	$-$	$-$	$-$	$-$	$-$	$-$	$+$	$-$	$-$	$\approx$	$+$	$-$	$-$	$\approx$	$\approx$
$\lambda_{1\gamma}^\alpha$	$-$	$+$	$+$	$\approx$	$\approx$	$+$	$+$	$+$	$\approx$	$+$	$+$	$+$	$+$	$\approx$	$-$	$-$	$-$	$+$
$\lambda_{1\delta}^\beta$	$+$	$-$	$-$	$\approx$	$\approx$	$\approx$	$\approx$	$\approx$	$\approx$	$-$	$+$	$+$	$\approx$	$+$	$+$	$+$	$+$	$+$
$\lambda_{1k}^\gamma$	$+$	$-$	$-$	$\approx$	$\approx$	$\approx$	$\approx$	$\approx$	$\approx$	$-$	$+$	$+$	$\approx$	$-$	$+$	$+$	$\approx$	$\approx$
$\lambda_{1k}^\delta$	$\approx$	$\approx$	$\approx$	$\approx$	$-$	$+$	$+$	$+$	$+$	$\approx$	$\approx$	$\approx$	$\approx$	$\approx$	$\approx$	$\approx$	$\approx$	$\approx$
$e_0^\phi$	$-$	$+$	$+$	$\approx$	$\approx$	$\approx$	$\approx$	$\approx$	$\approx$	$+$	$+$	$+$	$+$	$+$	$+$	$+$	$+$	$+$
$e_1^\alpha$	$-$	$-$	$-$	$-$	$\approx$	$\approx$	$\approx$	$\approx$	$-$	$+$	$+$	$+$	$+$	$-$	$+$	$+$	$-$	$\approx$
$e_1^\beta$	$+$	$-$	$-$	$\approx$	$-$	$-$	$-$	$-$	$\approx$	$-$	$+$	$+$	$\approx$	$\approx$	$+$	$+$	$+$	$\approx$
$e_1^\gamma$	$+$	$-$	$-$	$+$	$\approx$	$\approx$	$\approx$	$\approx$	$+$	$-$	$+$	$+$	$\approx$	$-$	$+$	$+$	$\approx$	$\approx$
$e_1^\delta$	$\approx$	$\approx$	$\approx$	$\approx$	$+$	$+$	$+$	$+$	$+$	$\approx$	$\approx$	$\approx$	$\approx$	$\approx$	$\approx$	$\approx$	$\approx$	$\approx$
$c_1^\alpha$	$-$	$-$	$-$	$-$	$\approx$	$\approx$	$\approx$	$\approx$	$-$	$+$	$+$	$+$	$+$	$-$	$+$	$+$	$-$	$\approx$
Note: $k^\gamma \equiv (k_1^\gamma + k_2^\gamma + k_3^\gamma)/3, k^\delta \equiv (k_1^\delta + k_2^\delta + k_3^\delta)/3, k \equiv (k^\gamma + k^\delta)/2$																		
$v_\gamma^\alpha \equiv (v_{1\gamma}^\alpha + v_{2\gamma}^\alpha + v_{3\gamma}^\alpha)/3, v_\delta^\beta \equiv (v_{1\delta}^\beta + v_{2\delta}^\beta + v_{3\delta}^\beta)/3, v \equiv (v_\gamma^\alpha + v_\delta^\beta)/2$																		

Table III (continue)															
	$b_0^\alpha$	$q_1^\alpha$	$q_2^\alpha$	$q_3^\alpha$	$b_0^\beta$	$q_1^\beta$	$q_2^\beta$	$q_3^\beta$	$q_0^\phi$	$b_1^\phi$	$b_2^\phi$	$b_3^\phi$	$b_j^\phi$	$b_j^\delta$	$q_j^\gamma$
$M$	$+$ $\approx$	$+$ $\approx$	$\approx$	$\approx$	$+$ $\approx$	$+$ $\approx$	$\approx$	$\approx$	$\approx$	$+$ $\approx$	$+$ $\approx$	$+$ $\approx$	$+$	$-$ $\approx$	$\approx$
$m_1^\alpha$	$+$ $\approx$	$+$ $\approx$	$-$ $\approx$	$-$ $\approx$	$-$ $\approx$	$+$ $\approx$	$-$ $\approx$	$-$ $\approx$	$\approx$	$+$ $\approx$	$-$ $\approx$	$-$ $\approx$	$+$	$-$ $\approx$	$+$ $\approx$
$\overline{w}$	$+$	$+$	$-$	$-$	$-$	$+$	$-$	$-$	$\approx$	$+$	$-$	$-$	$+$	$+$	$+/-$
$\lambda_{1\gamma}^\alpha$	$+$ $\approx$	$+$	$-$ $\approx$	$-$ $\approx$	$-$ $\approx$	$+$ $\approx$	$-$ $\approx$	$-$ $\approx$	$\approx$	$+$	$-$	$-$	$+$	$-$	$+$ $\approx$
$\lambda_{1\delta}^\beta$	$-$ $\approx$	$-$	$+$ $\approx$	$+$ $\approx$	$+$ $\approx$	$+$ $\approx$	$+$ $\approx$	$+$ $\approx$	$-$ $\approx$	$-$	$+$	$+$	$-$	$+/-$	$-$
$\lambda_{1k}^\gamma$	$-$	$-$ $\approx$	$+$ $\approx$	$+$ $\approx$	$+$	$-$ $\approx$	$+$ $\approx$	$+$ $\approx$	$\approx$	$-$	$+$	$+$	$-$	$+$	$-$ $\approx$
$\lambda_{1k}^\delta$	$\approx$	$\approx$	$\approx$	$\approx$	$\approx$	$\approx$	$\approx$	$\approx$	$\approx$	$\approx$	$\approx$	$\approx$	$\approx$	$+$	$+$
$e_0^\phi$	$-$ $\approx$	$+$ $\approx$	$-$ $\approx$	$-$ $\approx$	$+$ $\approx$	$+$ $\approx$	$-$ $\approx$	$-$ $\approx$	$+$	$+$	$-$	$-$	$+$	$-$	$-$ $\approx$
$e_1^\alpha$	$+$	$+$	$+$ $\approx$	$+$ $\approx$	$-$	$-$	$+$ $\approx$	$+$ $\approx$	$-$	$-$	$+$	$+$	$-$	$+$	$+$
$e_1^\beta$	$-$	$-$	$+$ $\approx$	$+$ $\approx$	$+$	$+$	$+$ $\approx$	$+$ $\approx$	$-$	$-$	$+$	$+$	$-$	$+/-$	$-$
$e_1^\gamma$	$-$	$-$ $\approx$	$+$ $\approx$	$+$ $\approx$	$+$	$-$ $\approx$	$+$ $\approx$	$+$ $\approx$	$\approx$	$-$	$+$	$+$	$-$	$+$	$-$
$e_1^\delta$	$\approx$	$\approx$	$\approx$	$\approx$	$\approx$	$\approx$	$\approx$	$\approx$	$\approx$	$\approx$	$\approx$	$\approx$	$\approx$	$+$	$+$
$c_1^\alpha$	$+$	$+$	$+$ $\approx$	$+$ $\approx$	$-$	$-$	$+$ $\approx$	$+$ $\approx$	$-$	$-$	$+$	$+$	$-$	$+$	$+$

Table III (continue)							
	$U^\alpha$	$U^\beta$	$U^\phi$	$U^\gamma$	$U^\delta$	$U^H$	$U^B$
$M$	$+$ $\approx$	$\approx$	$\approx$	$+$ $\approx$	$-$ $\approx$	$\approx$	$-$ $\approx$
$m_1^\alpha$	$+$ $\approx$	$\approx$	$\approx$	$+$ $\approx$	$+$ $\approx$	$\approx$	$+$ $\approx$
$\overline{w}$	$+$ $\approx$	$-$ $\approx$	$+/-$	$-$	$+$ $\approx$	$\approx$	$-$
$\lambda_{1\gamma}^\alpha$	$+$	$-$ $\approx$	$+$ $\approx$	$+$	$-$ $\approx$	$+$ $\approx$	$\approx$
$\lambda_{1\delta}^\beta$	$-$ $\approx$	$+$ $\approx$	$+$ $\approx$	$-$	$+$ $\approx$	$+$ $\approx$	$\approx$
$\lambda_{1k}^\gamma$	$-$ $\approx$	$+$ $\approx$	$+/-$	$-$	$-$	$\approx$	$-$
$\lambda_{1k}^\delta$	$\approx$	$\approx$	$\approx$	$\approx$	$-$	$\approx$	$-$
$e_0^\phi$	$+$	$+$	$+$	$+$	$-$ $\approx$	$+$	$\approx$
$e_1^\alpha$	$+$	$-$	$+$	$-$	$+$ $\approx$	$+$ $\approx$	$-$ $\approx$
$e_1^\beta$	$-$	$+$ $\approx$	$+$	$-$	$+$ $\approx$	$+$ $\approx$	$\approx$
$e_1^\gamma$	$\approx$	$\approx$	$\approx$	$+$	$\approx$	$\approx$	$+$ $\approx$
$e_1^\delta$	$\approx$	$\approx$	$\approx$	$\approx$	$+$ $\approx$	$\approx$	$+$ $\approx$
$c_1^\alpha$	$-$	$-$	$+$	$-$	$+$ $\approx$	$-$ $\approx$	$-$ $\approx$
Note: $U^H \equiv (U^\alpha + U^\beta + U^\phi)$ $U^B \equiv U^\gamma + U^\delta$							

## 6 Appendix II

We only describe the problems for Mr.  $\phi$  and the two banks since those of Mr.  $\alpha$  and  $\beta$  remain the same as in the baseline model.

### 6.1 Household $\phi$

$$\max_{\{q_0^\phi, b_s^\phi, d_\gamma^\phi, d_\delta^\phi\}, s \in S} U^\phi = [\chi_0^\phi - c_\phi(\chi_0^\phi)^2] + \sum_{s \in S} [\chi_s^\phi - c_{\phi,s}(\chi_s^\phi)^2]$$

subject to

$$d_\delta^\phi + d_\gamma^\phi \leq m_0^\phi \quad (\text{A1})$$

$$q_0^\phi \leq e_0^\phi \quad (\text{A2})$$

$$\chi_0^\phi \leq e_0^\phi - q_0^\phi \quad (\text{A3})$$

$$b_s^\phi \leq \Delta(\text{A1}) + p_0 q_0^\phi + v_{s\phi}^\gamma d_\gamma^\phi (1 + r_d^\gamma) + d_\delta^\phi (1 + r_d^\delta), \quad s \in S \quad (\text{A4})$$

$$\chi_s^\phi \leq \frac{b_s^\phi}{p_s}, \quad s \in S \quad (\text{A5})$$

where,

$d_b^\phi \equiv$  amount of money that Mr.  $\phi$  deposits with bank  $b$ ,  $b \in B$

$r_d^b \equiv$  deposit rate offered by bank  $b \in B$ , and

$v_{s\phi}^\gamma \equiv$  bank  $\gamma$ 's repayment rates in the deposit market in state  $s \in S$ .

### 6.2 Bank $\gamma$

$$\max_{\{\mu^\gamma, \mu_d^\gamma, \bar{m}^\gamma, v_s^\gamma, v_{s\phi}^\gamma\}, s \in S} U^\gamma = \sum_{s \in S} [\pi_s^\gamma] - \sum_{s \in S} \left[ \lambda_{ks}^\gamma \max[0, \bar{k} - k_s^\gamma] - \lambda_s^\gamma [\mu^\gamma - v_s^\gamma \mu^\gamma]^2 - \lambda_{s\phi}^\gamma [\mu_d^\gamma - v_{s\phi}^\gamma \mu_d^\gamma]^2 \right]$$

subject to

$$\bar{m}^\gamma \leq \frac{\mu^\gamma}{(1 + \rho)} + \frac{\mu_d^\gamma}{(1 + r_d^\gamma)} \quad (\text{A6})$$

$$v_s^\gamma \mu^\gamma + v_{s\phi}^\gamma \mu_d^\gamma \leq \Delta(\text{A6}) + v_{s\gamma}^\alpha (1 + r^\gamma) \bar{m}^\gamma + e_s^\gamma, \quad s \in S \quad (\text{A7})$$

where,

$\pi_s^\gamma = \Delta(\text{A7})$ ,

$k_s^\gamma = \frac{e_s^\gamma}{\bar{\omega} v_{s\gamma}^\alpha (1 + r^\gamma) \bar{m}^\gamma}, \quad s \in S$ ,

$v_s^\gamma \equiv$  bank  $\gamma$ 's repayment rate in the interbank market in state  $s \in S$ ,

$\mu_d^b \equiv$  deposit demand by bank  $b$ ,  $b \in B$ ,

$\lambda_s^\gamma \equiv$  default penalty in the interbank market imposed on bank  $\gamma$  in state  $s \in S$ , and

$\lambda_{s\phi}^\gamma \equiv$  default penalty in the deposit market imposed on bank  $\gamma$  in state  $s \in S$

### 6.3 Bank $\delta$

$$\max_{\{d^\delta, \mu_d^\delta, \bar{m}^\delta\}} U^\delta = \sum_{s \in S} [\pi_s^\delta] - \sum_{s \in S} \lambda_{ks}^\delta \max[0, \bar{k} - k_s^\delta]$$

subject to

$$d^\delta \leq e_0^\delta \quad (\text{A8})$$

$$\bar{m}^\delta \leq \Delta(\text{A8}) + \frac{\mu_d^\delta}{(1 + r_d^\delta)} \quad (\text{A9})$$

$$\mu_d^\delta \leq \Delta(\text{A9}) + v_{s\delta}^\beta \bar{m}^\delta (1 + r^\delta) + v_s^\gamma d^\delta (1 + \rho) + e_s^\delta \quad (\text{A10})$$

where,

$\pi_s^\delta = \Delta(\text{A10})$ , and

$$k_s^\delta = \frac{e_1^\delta}{\bar{\omega} v_{s\delta}^\beta (1 + r^\delta) \bar{m}^\delta + \omega v_s^\gamma d^\delta (1 + \rho)}.$$

### 6.4 Market Clearing Conditions

$$p_0 = \frac{b_0^\alpha + b_0^\beta}{q_0^\phi} \quad (\text{i.e. commodity market at } t = 1 \text{ clears}) \quad (\text{A11})$$

$$p_s = \frac{b_s^\phi}{q_s^\alpha + q_s^\beta}, \quad s \in S \quad (\text{i.e. commodity market at } t = 2, \quad s \in S \text{ clears}) \quad (\text{A12})$$

$$1 + \rho = \frac{\mu^\gamma}{M^{CB} + d^\delta} \quad (\text{i.e. interbank market clears}) \quad (\text{A13})$$

$$1 + r^\gamma = \frac{\mu^{\alpha\gamma}}{\bar{m}^\gamma} \quad (\text{i.e. bank } \gamma\text{'s loan market clears}) \quad (\text{A14})$$

$$1 + r^\delta = \frac{\mu^{\beta\delta}}{\bar{m}^\delta} \quad (\text{i.e. bank } \delta\text{'s loan market clears}) \quad (\text{A15})$$

$$1 + r_d^\gamma = \frac{\mu_d^\gamma}{d_\gamma^\phi} \quad (\text{i.e. bank } \gamma\text{'s deposit market clears}) \quad (\text{A16})$$

$$1 + r_d^\delta = \frac{\mu_d^\delta}{d_\delta^\phi} \quad (\text{i.e. bank } \delta\text{'s deposit market clears}) \quad (\text{A17})$$

Equilibrium is defined similarly to that given in section 2.1.6.

## 7 Appendix III

Table I: Exogenous variables						
Coefficient of risk aversion	Endowment			Penalty		Others
	Commodities	Money	Capital	Default	CAR violation	
$c_0^\alpha = 0.011118$	$e_0^\alpha = 0$	$m_0^\alpha = 0$	$e_0^\gamma = 9$	$\lambda_{1\gamma}^\alpha = 0.2$	$\lambda_{1k}^\gamma = 3$	$M^{CB} = 0.5$ $\bar{\omega} = 1$ $\omega = 0.5$ $\bar{k} = 0.1$
$c_1^\alpha = 0.111429$	$e_1^\alpha = 27$	$m_1^\alpha = 2.9472$	$e_1^\gamma = 1.7278$	$\lambda_{2\gamma}^\alpha = 0.1$	$\lambda_{2k}^\gamma = 2$	
$c_2^\alpha = 0.073333$	$e_2^\alpha = 27$	$m_2^\alpha = 3.015$	$e_2^\gamma = 1.4108$	$\lambda_{3\gamma}^\alpha = 0.05$	$\lambda_{3k}^\gamma = 1$	
$c_3^\alpha = 0.066786$	$e_3^\alpha = 27$	$m_3^\alpha = 1.9015$	$e_3^\gamma = 1.1161$	$\lambda_{1\delta}^\beta = 0.2$	$\lambda_{1k}^\delta = 4$	
$c_0^\beta = 0.010476$	$e_0^\beta = 0$	$m_0^\beta = 0$	$e_0^\delta = 21.5$	$\lambda_{2\delta}^\beta = 0.1$	$\lambda_{2k}^\delta = 2$	
$c_1^\beta = 0.014740$	$e_1^\beta = 27$	$m_1^\beta = 3.3973$	$e_1^\delta = 2.0474$	$\lambda_{3\delta}^\beta = 0.05$	$\lambda_{3k}^\delta = 1$	
$c_2^\beta = 0.105184$	$e_2^\beta = 27$	$m_2^\beta = 1.8518$	$e_2^\delta = 1.6555$	$\lambda_1^\gamma = 1.576$		
$c_3^\beta = 0.070840$	$e_3^\beta = 27$	$m_3^\beta = 2.5431$	$e_3^\delta = 1.3160$	$\lambda_2^\gamma = 1.123$		
$c_0^\phi = 0.040000$	$e_0^\phi = 45$	$m_0^\phi = 9$		$\lambda_3^\gamma = 0.936$		
$c_1^\phi = 0.007000$	$e_1^\phi = 0$	$m_1^\phi = 0$		$\lambda_{1\phi}^\gamma = 1.685$		
$c_2^\phi = 0.008673$	$e_2^\phi = 0$	$m_2^\phi = 0$		$\lambda_{2\phi}^\gamma = 1.404$		
$c_3^\phi = 0.010767$	$e_3^\phi = 0$	$m_3^\phi = 0$		$\lambda_{3\phi}^\gamma = 1.203$		

Table II: Initial Equilibrium					
Prices	Interest rates	Loans/deposits	Capital/Asset ratio	Repayment rate	Commodities
$p_0 = 1$	$r^\gamma = 0.65$	$\bar{m}^\gamma = 19.05$	$k_1^\gamma = 0.06$	$v_{1\gamma}^\alpha = 0.919$	$b_0^\alpha = 19.04$
$p_1 = 1.1$	$r^\delta = 0.6$	$\bar{m}^\delta = 21$	$k_2^\gamma = 0.05$	$v_{2\gamma}^\alpha = 0.9$	$q_1^\alpha = 23.5$
$p_2 = 1.2$	$\rho = 0.48$	$d_\gamma^\phi = 4.31$	$k_3^\gamma = 0.04$	$v_{3\gamma}^\alpha = 0.89$	$q_2^\alpha = 21$
$p_3 = 1.3$	$r_d^\gamma = 0.48$	$d_\delta^\phi = 4.69$	$k_1^\delta = 0.06$	$v_{1\delta}^\beta = 0.899$	$q_3^\alpha = 20$
	$r_d^\delta = 0.4$	$d^\delta = 5.24$	$k_2^\delta = 0.05$	$v_{2\delta}^\beta = 0.87$	$b_0^\beta = 20.95$
		$\mu^{\alpha\gamma} = 31.37$	$k_3^\delta = 0.04$	$v_{3\delta}^\beta = 0.866$	$q_1^\beta = 24.355$
		$\mu^{\beta\delta} = 33.58$		$v_1^\gamma = 0.963$	$q_2^\beta = 22.82$
		$\mu^\gamma = 8.5$		$v_2^\gamma = 0.95$	$q_3^\beta = 20.4$
		$\mu_d^\gamma = 6.38$		$v_3^\gamma = 0.94$	$q_0^\phi = 40$
		$\mu_d^\delta = 6.58$		$v_{1\phi}^\gamma = 0.954$	$b_1^\phi = 52.67$
				$v_{2\phi}^\gamma = 0.944$	$b_2^\phi = 52.61$
				$v_{3\phi}^\gamma = 0.935$	$b_3^\phi = 52.55$



Table III: Directional Effects of an increase in exogenous parameters on endogenous variables																			
	$p_0$	$p_1$	$p_2$	$p_3$	$r^\gamma$	$r^\delta$	$\rho$	$r_d^\gamma$	$r_d^\delta$	$\overline{m}^\gamma$	$\overline{m}^\delta$	$d_\gamma^\phi$	$d_\delta^\phi$	$\mu_d^\gamma$	$\mu_d^\delta$	$d^\delta$	$\mu^{\alpha\gamma}$	$\mu^{\beta\delta}$	$\mu^\gamma$
$M^{CB}$	+	+	+	+	-	-	-	-	-	+	+	+	-	+	-	-	$\approx$	$\approx$	+
$m_1^\alpha$	$\approx$	+	+	+	-	+	+	+	+	+	-	-	-	+	-	+	+	-	+
$\overline{w}$	$\approx$	+	+	+	-	+	+	+	+	+	-	+	-	+	-	-	$\approx$	$\approx$	-
$\lambda_{1\gamma}^\alpha$	+	-	$\approx$	$\approx$	-	+	-	-	+	+	-	+	-	+	-	+	-	-	+
$\lambda_{1\delta}^\beta$	+	-	$\approx$	$\approx$	+	-	+	+	-	-	+	-	+	-	+	-	-	-	-
$\lambda_{1k}^\gamma$	$\approx$	-	-	-	+	-	-	-	-	-	+	-	+	-	+	-	-	+	-
$\lambda_{1k}^\delta$	$\approx$	-	-	-	-	+	-	-	-	+	-	-	+	-	+	+	+	-	+
$e_0^\phi$	-	$\approx$	$\approx$	$\approx$	+	+	$\approx$	$\approx$	$\approx$	$\approx$	$\approx$	$\approx$	$\approx$	$\approx$	$\approx$	$\approx$	+	+	$\approx$
$e_1^\alpha$	+	-	$\approx$	$\approx$	-	+	-	-	-	+	-	+	-	+	-	+	+	-	+
$e_1^\beta$	+	-	$\approx$	$\approx$	+	-	+	+	-	-	+	-	+	-	+	-	-	+	-
$e_1^\gamma$	$\approx$	-	-	-	+	-	-	-	-	-	+	-	+	-	+	-	-	+	-
$e_1^\delta$	$\approx$	-	-	-	-	+	-	-	-	+	-	-	+	-	+	+	+	-	+
$c_1^\alpha$	+	-	$\approx$	$\approx$	-	+	-	-	+	+	-	-	+	+	-	+	+	-	+
$e_0^\delta$	+	+	+	+	-	-	-	-	-	+	+	+	-	+	-	+	+	$\approx$	+
$e_0^\gamma$	+	+	+	+	-	-	-	-	-	+	+	-	+	-	+	-	$\approx$	+	-
$\lambda_1^\gamma$	$\approx$	+	+	+	-	+	-	-	+	+	-	+	-	+	-	-	+	-	-
$\lambda_{1\phi}^\gamma$	$\approx$	+	-	-	-	-	-	-	+	+	-	-	+	-	+	+	+	-	+
$m_1^\beta$	+	+	+	+	+	-	+	+	+	-	+	-	+	-	+	-	-	+	-
$m_0^\phi$	+	+	+	+	-	-	-	-	-	+	+	+	-	+	-	+	+	$\approx$	+
Note: +(-) : substantial increase (decrease)																			
$\approx$ +(-) : weak increase (decrease), $\approx$ : approximately equal																			
+/- : ambiguous effect																			

Table III (continue)																	
	$v_{1\gamma}^\alpha$	$v_{2\gamma}^\alpha$	$v_{3\gamma}^\alpha$	$v_\gamma^\alpha$	$v_{1\delta}^\beta$	$v_{2\delta}^\beta$	$v_{3\delta}^\beta$	$v_\delta^\beta$	$v$	$v_1^\gamma$	$v_2^\gamma$	$v_3^\gamma$	$v^\gamma$	$v_{1\phi}^\gamma$	$v_{2\phi}^\gamma$	$v_{3\phi}^\gamma$	$v_\phi^\gamma$
$M^{CB}$	$\approx +$	$\approx +$	$\approx +$	$\approx +$	$\approx +$	$\approx +$	$\approx +$	$\approx +$	$\approx +$	$\approx +$	$\approx +$	$\approx +$	$\approx +$	$\approx +$	$\approx +$	$\approx +$	$\approx +$
$m_1^\alpha$	$\approx +$	$\approx +$	$\approx +$	$\approx +$	$\approx +$	$\approx +$	$\approx +$	$\approx +$	$\approx +$	$\approx +$	$\approx +$	$\approx +$	$\approx +$	$\approx +$	$\approx +$	$\approx +$	$\approx +$
$\bar{w}$	$\approx +$	$\approx +$	$\approx +$	$\approx +$	$\approx +$	$\approx +$	$\approx +$	$\approx +$	$\approx +$	$\approx -$	$\approx -$	$\approx -$	$\approx -$	$\approx +$	$\approx +$	$\approx +$	$\approx +$
$\lambda_{1\gamma}^\alpha$	$\approx +$	$\approx +$	$\approx +$	$\approx +$	$\approx -$	$\approx +$	$\approx +$	$\approx -$	$\approx +$	$\approx +$	$\approx +$	$\approx +$	$\approx +$	$\approx +$	$\approx +$	$\approx +$	$\approx +$
$\lambda_{1\delta}^\beta$	$\approx -$	$\approx +$	$\approx +$	$\approx -$	$\approx +$	$\approx +$	$\approx +$	$\approx +$	$\approx +$	$\approx -$	$\approx -$	$\approx -$	$\approx -$	$\approx -$	$\approx -$	$\approx -$	$\approx -$
$\lambda_{1k}^\gamma$	$\approx +$	$\approx +$	$\approx +$	$\approx +$	$\approx -$	$\approx -$	$\approx -$	$\approx -$	$\approx -$	$\approx -$	$\approx -$	$\approx -$	$\approx -$	$\approx -$	$\approx -$	$\approx -$	$\approx -$
$\lambda_{1k}^\delta$	$\approx -$	$\approx -$	$\approx -$	$\approx -$	$\approx -$	$\approx -$	$\approx -$	$\approx -$	$\approx -$	$\approx +$	$\approx +$	$\approx +$	$\approx +$	$\approx -$	$\approx -$	$\approx -$	$\approx -$
$e_0^\phi$	$\approx +$	$\approx +$	$\approx +$	$\approx +$	$\approx +$	$\approx +$	$\approx +$	$\approx +$	$\approx +$	$\approx +$	$\approx +$	$\approx +$	$\approx +$	$\approx +$	$\approx +$	$\approx +$	$\approx +$
$e_1^\alpha$	$\approx +$	$\approx -$	$\approx -$	$\approx +$	$\approx -$	$\approx +$	$\approx +$	$\approx -$	$\approx +$	$\approx +$	$\approx +$	$\approx +$	$\approx +$	$\approx +$	$\approx +$	$\approx +$	$\approx +$
$e_1^\beta$	$\approx -$	$\approx +$	$\approx +$	$\approx -$	$\approx +$	$\approx -$	$\approx -$	$\approx +$	$\approx +$	$\approx -$	$\approx -$	$\approx -$	$\approx -$	$\approx -$	$\approx -$	$\approx -$	$\approx -$
$e_1^\gamma$	$\approx +$	$\approx +$	$\approx +$	$\approx +$	$\approx -$	$\approx -$	$\approx -$	$\approx -$	$\approx -$	$\approx -$	$\approx -$	$\approx -$	$\approx -$	$\approx -$	$\approx -$	$\approx -$	$\approx -$
$e_1^\delta$	$\approx -$	$\approx -$	$\approx -$	$\approx -$	$\approx -$	$\approx -$	$\approx -$	$\approx -$	$\approx -$	$\approx +$	$\approx +$	$\approx +$	$\approx +$	$\approx -$	$\approx -$	$\approx -$	$\approx -$
$c_1^\alpha$	$\approx +$	$\approx -$	$\approx -$	$\approx +$	$\approx -$	$\approx +$	$\approx +$	$\approx -$	$\approx +$	$\approx +$	$\approx +$	$\approx +$	$\approx +$	$\approx +$	$\approx +$	$\approx +$	$\approx +$
$e_0^\delta$	$\approx +$	$\approx +$	$\approx +$	$\approx +$	$\approx +$	$\approx +$	$\approx +$	$\approx +$	$\approx +$	$\approx +$	$\approx +$	$\approx +$	$\approx +$	$\approx +$	$\approx +$	$\approx +$	$\approx +$
$e_0^\gamma$	$\approx +$	$\approx +$	$\approx +$	$\approx +$	$\approx +$	$\approx +$	$\approx +$	$\approx +$	$\approx +$	$\approx -$	$\approx -$	$\approx -$	$\approx -$	$\approx -$	$\approx -$	$\approx -$	$\approx -$
$\lambda_1^\gamma$	$\approx +$	$\approx +$	$\approx +$	$\approx +$	$\approx +$	$\approx +$	$\approx +$	$\approx +$	$\approx +$	$\approx +$	$\approx -$	$\approx -$	$\approx +$	$\approx +$	$\approx +$	$\approx +$	$\approx +$
$\lambda_{1\phi}^\gamma$	$\approx +$	$\approx -$	$\approx -$	$\approx -$	$\approx +$	$\approx +$	$\approx +$	$\approx +$	$\approx +$	$\approx +$	$\approx +$	$\approx +$	$\approx +$	$\approx +$	$\approx -$	$\approx -$	$\approx +$
$m_1^\beta$	$\approx +$	$\approx +$	$\approx +$	$\approx +$	$\approx +$	$\approx +$	$\approx +$	$\approx +$	$\approx +$	$\approx -$	$\approx -$	$\approx -$	$\approx -$	$\approx -$	$\approx -$	$\approx -$	$\approx -$
$m_0^\phi$	$\approx +$	$\approx +$	$\approx +$	$\approx +$	$\approx +$	$\approx +$	$\approx +$	$\approx +$	$\approx +$	$\approx +$	$\approx +$	$\approx +$	$\approx +$	$\approx +$	$\approx +$	$\approx +$	$\approx +$
Note: $v_\gamma^\alpha \equiv (v_{1\gamma}^\alpha + v_{2\gamma}^\alpha + v_{3\gamma}^\alpha)/3$ , $v_\delta^\beta \equiv (v_{1\delta}^\beta + v_{2\delta}^\beta + v_{3\delta}^\beta)/3$ , $v \equiv (v_\gamma^\alpha + v_\delta^\beta)/2$																	
$v^\gamma \equiv (v_1^\gamma + v_2^\gamma + v_3^\gamma)/3$ , $v_\phi^\gamma \equiv (v_{1\phi}^\gamma + v_{2\phi}^\gamma + v_{3\phi}^\gamma)/3$																	

Table III (continue)												
	$b_0^\alpha$	$q_1^\alpha$	$q_2^\alpha$	$q_3^\alpha$	$b_0^\beta$	$q_1^\beta$	$q_2^\beta$	$q_3^\beta$	$q_0^\phi$	$b_1^\phi$	$b_2^\phi$	$b_3^\phi$
$M^{CB}$	$+$ $\approx$	$\approx$	$\approx$	$\approx$	$+$	$\approx$	$\approx$	$\approx$	$\approx$	$+$ $\approx$	$+$ $\approx$	$+$ $\approx$
$m_1^\alpha$	$+$ $\approx$	$\approx$	$\approx$	$\approx$	$-$ $\approx$	$\approx$	$\approx$	$\approx$	$\approx$	$+$ $\approx$	$+$ $\approx$	$+$ $\approx$
$\overline{w}$	$+$ $\approx$	$\approx$	$\approx$	$\approx$	$-$ $\approx$	$\approx$	$\approx$	$\approx$	$\approx$	$+$ $\approx$	$+$ $\approx$	$+$ $\approx$
$\lambda_{1\gamma}^\alpha$	$+$ $\approx$	$+$	$\approx$	$\approx$	$-$ $\approx$	$-$ $\approx$	$\approx$	$\approx$	$-$ $\approx$	$\approx$	$\approx$	$\approx$
$\lambda_{1\delta}^\beta$	$-$ $\approx$	$-$ $\approx$	$\approx$	$\approx$	$+$ $\approx$	$+$ $\approx$	$\approx$	$\approx$	$-$ $\approx$	$\approx$	$\approx$	$\approx$
$\lambda_{1k}^\gamma$	$-$ $\approx$	$\approx$	$\approx$	$\approx$	$+$ $\approx$	$\approx$	$\approx$	$\approx$	$\approx$	$-$ $\approx$	$-$ $\approx$	$-$ $\approx$
$\lambda_{1k}^\delta$	$+$ $\approx$	$\approx$	$\approx$	$\approx$	$-$ $\approx$	$\approx$	$\approx$	$\approx$	$\approx$	$-$ $\approx$	$-$ $\approx$	$-$ $\approx$
$e_0^\phi$	$\approx$	$\approx$	$\approx$	$\approx$	$\approx$	$\approx$	$\approx$	$\approx$	$+$	$\approx$	$\approx$	$\approx$
$e_1^\alpha$	$+$	$+$	$\approx$	$\approx$	$-$	$-$ $\approx$	$\approx$	$\approx$	$-$	$-$	$+$	$+$
$e_1^\beta$	$-$	$-$ $\approx$	$\approx$	$\approx$	$+$	$+$	$\approx$	$\approx$	$-$ $\approx$	$\approx$	$\approx$	$\approx$
$e_1^\gamma$	$-$ $\approx$	$\approx$	$\approx$	$\approx$	$+$ $\approx$	$\approx$	$\approx$	$\approx$	$\approx$	$-$ $\approx$	$-$ $\approx$	$-$ $\approx$
$e_1^\delta$	$+$ $\approx$	$\approx$	$\approx$	$\approx$	$-$ $\approx$	$\approx$	$\approx$	$\approx$	$\approx$	$-$ $\approx$	$-$ $\approx$	$-$ $\approx$
$c_1^\alpha$	$+$	$+$	$\approx$	$\approx$	$-$	$-$ $\approx$	$\approx$	$\approx$	$-$ $\approx$	$\approx$	$\approx$	$\approx$
$e_0^\delta$	$+$	$+$ $\approx$	$+$ $\approx$	$+$ $\approx$	$+$	$+$ $\approx$	$+$ $\approx$	$+$ $\approx$	$+$ $\approx$	$+$	$+$	$+$
$e_0^\gamma$	$+$	$+$ $\approx$	$+$ $\approx$	$+$ $\approx$	$+$	$+$ $\approx$	$+$ $\approx$	$+$ $\approx$	$+$ $\approx$	$+$	$+$	$+$
$\lambda_1^\gamma$	$+$ $\approx$	$\approx$	$\approx$	$\approx$	$-$ $\approx$	$\approx$	$\approx$	$\approx$	$\approx$	$+$ $\approx$	$+$ $\approx$	$+$ $\approx$
$\lambda_{1\phi}^\gamma$	$+$ $\approx$	$\approx$	$\approx$	$\approx$	$-$ $\approx$	$\approx$	$\approx$	$\approx$	$\approx$	$+$ $\approx$	$-$ $\approx$	$-$ $\approx$
$m_1^\beta$	$-$	$\approx$	$\approx$	$\approx$	$+$	$\approx$	$\approx$	$\approx$	$-$ $\approx$	$+$ $\approx$	$+$ $\approx$	$+$ $\approx$
$m_0^\phi$	$+$	$+$ $\approx$	$+$ $\approx$	$+$ $\approx$	$+$	$+$ $\approx$	$+$ $\approx$	$+$ $\approx$	$+$ $\approx$	$+$	$+$	$+$

Table III (continue)																
	$U^\alpha$	$U^\beta$	$U^\phi$	$U^\gamma$	$U^\delta$	$U^H$	$U^B$	$k_1^\gamma$	$k_2^\gamma$	$k_3^\gamma$	$k^\gamma$	$k_1^\delta$	$k_2^\delta$	$k_3^\delta$	$k^\delta$	$k$
$M^{CB}$	$+$ $\approx$	$+$ $\approx$	$\approx$	$-$ $\approx$	$-$	$\approx$	$-$ $\approx$	$-$ $\approx$	$-$ $\approx$	$-$ $\approx$	$-$ $\approx$	$+$ $\approx$	$+$ $\approx$	$+$ $\approx$	$+$ $\approx$	$-$ $\approx$
$m_1^\alpha$	$+$	$-$ $\approx$	$\approx$	$+$ $\approx$	$+$ $\approx$	$+$ $\approx$	$+$ $\approx$	$-$	$-$	$-$	$-$	$-$	$-$	$-$	$-$	$-$
$\bar{\omega}$	$+$ $\approx$	$\approx$	$\approx$	$-$	$-$ $\approx$	$\approx$	$-$ $\approx$	$-$	$-$	$-$	$-$	$-$	$-$	$-$	$-$	$-$
$\lambda_{1\gamma}^\alpha$	$+$	$-$ $\approx$	$+$ $\approx$	$-$ $\approx$	$\approx$	$+$	$\approx$	$-$ $\approx$	$\approx$	$\approx$	$-$ $\approx$	$+$ $\approx$	$-$ $\approx$	$-$ $\approx$	$+$ $\approx$	$-$ $\approx$
$\lambda_{1\delta}^\beta$	$-$ $\approx$	$+$	$+$ $\approx$	$\approx$	$\approx$	$+$	$\approx$	$+$ $\approx$	$\approx$	$\approx$	$+$ $\approx$	$-$ $\approx$	$+$ $\approx$	$+$ $\approx$	$-$ $\approx$	$+$ $\approx$
$\lambda_{1k}^\gamma$	$-$ $\approx$	$+$ $\approx$	$\approx$	$+$ $\approx$	$-$ $\approx$	$\approx$	$-$ $\approx$	$+$ $\approx$	$+$ $\approx$	$+$ $\approx$	$+$ $\approx$	$-$ $\approx$	$-$ $\approx$	$-$ $\approx$	$-$ $\approx$	$+$ $\approx$
$\lambda_{1k}^\delta$	$+$ $\approx$	$-$ $\approx$	$\approx$	$-$ $\approx$	$-$ $\approx$	$\approx$	$-$ $\approx$	$+$ $\approx$	$+$ $\approx$	$+$ $\approx$	$+$ $\approx$	$-$	$+$	$+$	$+$	$\approx$
$e_0^\phi$	$+$	$+$	$+$ $\approx$	$\approx$	$\approx$	$+$	$\approx$	$\approx$	$\approx$	$\approx$	$\approx$	$\approx$	$\approx$	$\approx$	$\approx$	$\approx$
$e_1^\alpha$	$+$	$-$	$+$	$-$ $\approx$	$+$ $\approx$	$+$ $\approx$	$\approx$	$-$	$\approx$	$\approx$	$-$	$+$	$-$ $\approx$	$-$ $\approx$	$+$	$-$ $\approx$
$e_1^\beta$	$-$	$+$	$+$	$+$ $\approx$	$-$ $\approx$	$+$ $\approx$	$\approx$	$+$	$\approx$	$\approx$	$+$	$-$	$+$ $\approx$	$+$ $\approx$	$-$	$+$ $\approx$
$e_1^\gamma$	$-$ $\approx$	$+$ $\approx$	$\approx$	$+$	$-$ $\approx$	$\approx$	$+$	$+$	$+$ $\approx$	$+$ $\approx$	$+$ $\approx$	$+$ $\approx$	$+$ $\approx$	$+$ $\approx$	$+$ $\approx$	$+$
$e_1^\delta$	$+$ $\approx$	$-$ $\approx$	$\approx$	$-$	$+$	$\approx$	$+$ $\approx$	$+$ $\approx$	$+$ $\approx$	$+$ $\approx$	$+$ $\approx$	$+$	$-$ $\approx$	$-$ $\approx$	$+$	$+$
$c_1^\alpha$	$-$	$-$	$+$ $\approx$	$-$ $\approx$	$\approx$	$-$ $\approx$	$\approx$	$-$	$\approx$	$\approx$	$-$	$+$	$-$ $\approx$	$-$ $\approx$	$+$ $\approx$	$-$ $\approx$
$e_0^\delta$	$+$	$+$	$\approx$	$-$	$+$	$+$ $\approx$	$+$	$-$	$-$	$-$	$-$	$-$	$-$	$-$	$-$	$-$
$e_0^\gamma$	$+$ $\approx$	$+$ $\approx$	$\approx$	$+$	$-$	$+$ $\approx$	$+$	$-$	$-$	$-$	$-$	$-$ $\approx$	$-$ $\approx$	$-$ $\approx$	$-$ $\approx$	$-$
$\lambda_1^\gamma$	$+$ $\approx$	$-$ $\approx$	$\approx$	$-$ $\approx$	$+$ $\approx$	$\approx$	$+$ $\approx$	$-$ $\approx$	$-$ $\approx$	$-$ $\approx$	$-$ $\approx$	$+$ $\approx$	$+$ $\approx$	$+$ $\approx$	$+$ $\approx$	$+$ $\approx$
$\lambda_{1\phi}^\gamma$	$+$ $\approx$	$-$ $\approx$	$\approx$	$-$ $\approx$	$+$ $\approx$	$\approx$	$+$ $\approx$	$-$ $\approx$	$+$ $\approx$	$+$ $\approx$	$-$ $\approx$	$-$	$-$	$-$	$-$	$-$
$m_1^\beta$	$-$ $\approx$	$+$	$\approx$	$+$	$+$	$+$ $\approx$	$+$	$-$ $\approx$	$-$ $\approx$	$-$ $\approx$	$-$ $\approx$	$-$	$-$ $\approx$	$-$ $\approx$	$-$ $\approx$	$-$
$m_0^\phi$	$+$	$+$	$-$ $\approx$	$-$	$+$	$+$ $\approx$	$+$	$-$	$-$	$-$	$-$	$-$	$-$	$-$	$-$	$-$
Note: $U^H \equiv (U^\alpha + U^\beta + U^\phi)$ , $U^B \equiv U^\gamma + U^\delta$ $k^\gamma \equiv (k_1^\gamma + k_2^\gamma + k_3^\gamma)/3$ , $k^\delta \equiv (k_1^\delta + k_2^\delta + k_3^\delta)/3$ , $k \equiv (k^\gamma + k^\delta)/2$																

## 8 Glossary

$b_s^h$	amount of fiat money spent by $h$ to trade in the commodity market, $s = \{0\} \cup S$
$q_s^h$	amount of commodity offered for sales by $h$ , $s = \{0\} \cup S$
$\chi_s^h$	commodity consumption by $h$ in state $s \in S$
$\overline{m}^b$	amount of credit that bank $b$ extends
$\mu^{h^b}$	amount of fiat money agent $h^b \in \{\alpha^\gamma, \beta^\delta\}$ chooses to borrow from bank $b$
$\mu_d^b$	deposit demand by bank $b$
$d_b^\phi$	amount of money that Mr. $\phi$ deposits with bank $b$
$\mu^\gamma$	amount of money that bank $\gamma$ borrows from the interbank market
$d^\delta$	bank $\delta$ 's interbank deposits
$b_j^l$	amount of money placed by agent (bank) $l \in H \cup B$ in the Arrow security market
$q_j^\gamma$	bank $\gamma$ 's quantity supply of Arrow securities
$r_d^b$	deposit rate offered by bank $b$
$r^b$	lending rate offered by bank $b$
$\rho$	interbank rate
$p_s$	commodity price in $s = \{0\} \cup S$
$\theta$	asset price
$m_s^h$	monetary endowment of household $h$ in states $s = \{0\} \cup S$
$e_s^h$	commodity endowment of household $h$ in states $s = \{0\} \cup S$
$e_s^b$	initial capital endowment of bank $b$ in state $s = \{0\} \cup S$ ,
$\pi_s^b$	bank $b$ 's profit in state $s \in S$
$k_s^b$	capital to (risk-weighted) asset ratio of bank $b$ in state $s \in S$
$\bar{k}$	capital adequacy requirement set by the regulator
$\lambda_{sb}^h$	(non-pecuniary) default penalties in the consumer loan market imposed on $h$ in state $s \in S$
$\lambda_{ks}^b$	capital requirements' violation penalties on bank $b$ in state $s \in S$
$\lambda_s^\gamma(\lambda_{s\phi}^\gamma)$	default penalty in the interbank (deposit) market imposed on bank $\gamma$ in state $s \in S$
$\omega, \tilde{\omega}, \overline{\omega}$	risk weights for interbank market deposits, Arrow security, and consumer loans, respectively
$M^{CB}$	money supply
$c_s^l$	exogenous parameters in the utility/profit functions of agent (bank) $l$ where $l \in H \cup B$
$v_{sb}^h$	the rates of repayment in the loan market by household $h^b$ to bank $b$ in state $s \in S$
$v_s^\gamma(v_{s\phi}^\gamma)$	bank $\gamma$ 's repayment rate in the interbank (deposit) market in state $s \in S$

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